Lecture 29

Greedy: Activity-Selection Problem (contd.), MST

Greedy Algorithm for Activity-Selection

Greedy Algorithm for Activity-Selection

i	0	1	2	3	4	5	6	7	8	9	10	11	12
s_i	0	1	3	0	5	3	5	6	7	8	2	12	16
f_i	0	4	5	6	7	9	9	10	11	12	14	16	16

Greedy Algorithm for Activity-Selection

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Let's try to find $A_{0,12}$ using greedy choices!

Greedy Algorithm for Activity-Selection

$A_{0,12}$ ↓

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Greedy Algorithm for Activity-Selection

$A_{0,12}$

↓

$A_{0,1} \cup \{a_1\} \cup A_{1,12}$

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Greedy Algorithm for Activity-Selection

$$A_{0,12}$$

$\downarrow$

$$A_{0,1} \cup \{a_1\} \cup A_{1,12}$$

$\downarrow$

$$\varnothing$$

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Greedy Algorithm for Activity-Selection

$A_{0,12}$

$A_{0,1} \cup \{a_1\} \cup A_{1,12}$

$\emptyset$ $\quad$ $A_{1,4} \cup \{a_4\} \cup A_{4,12}$

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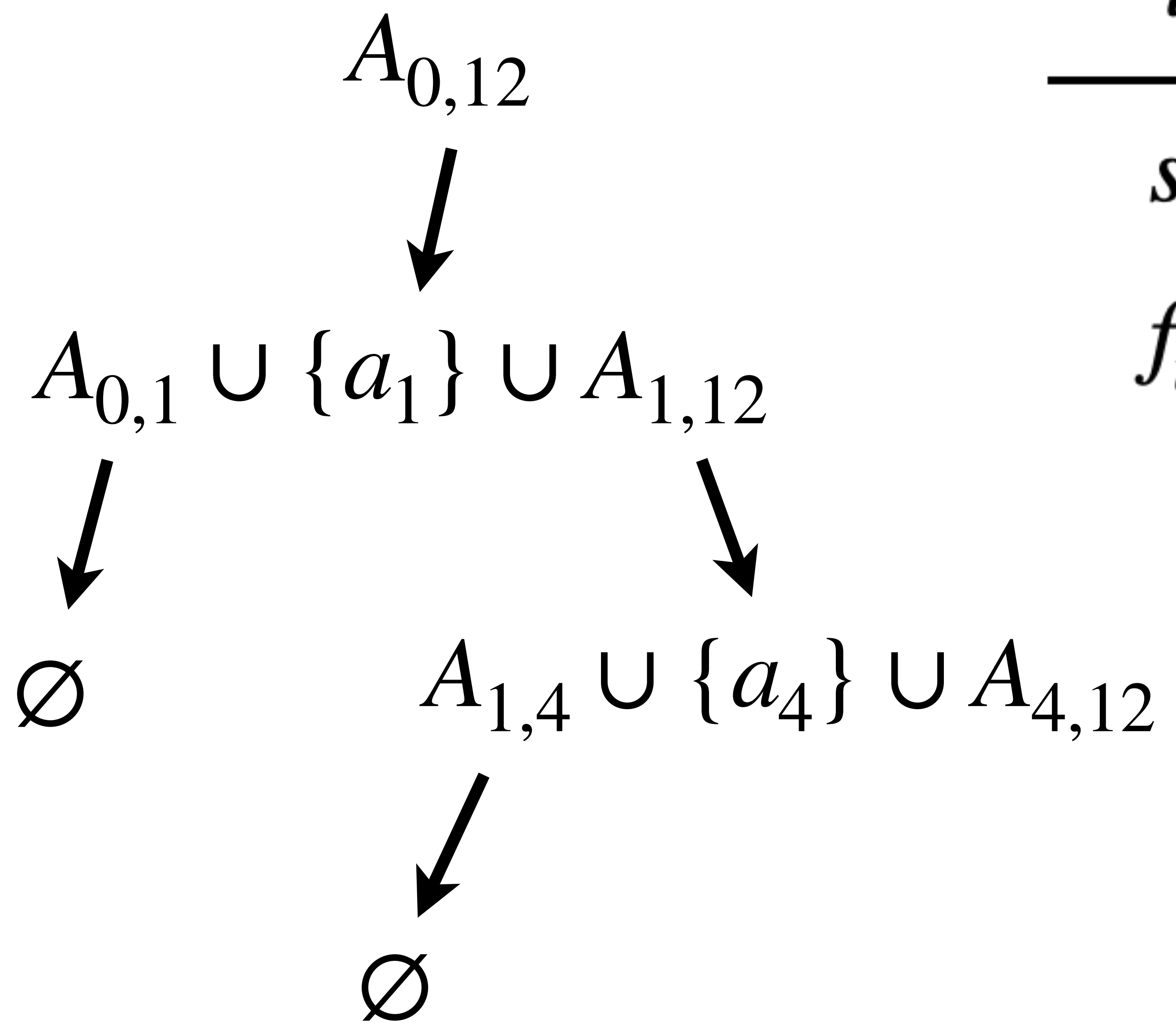

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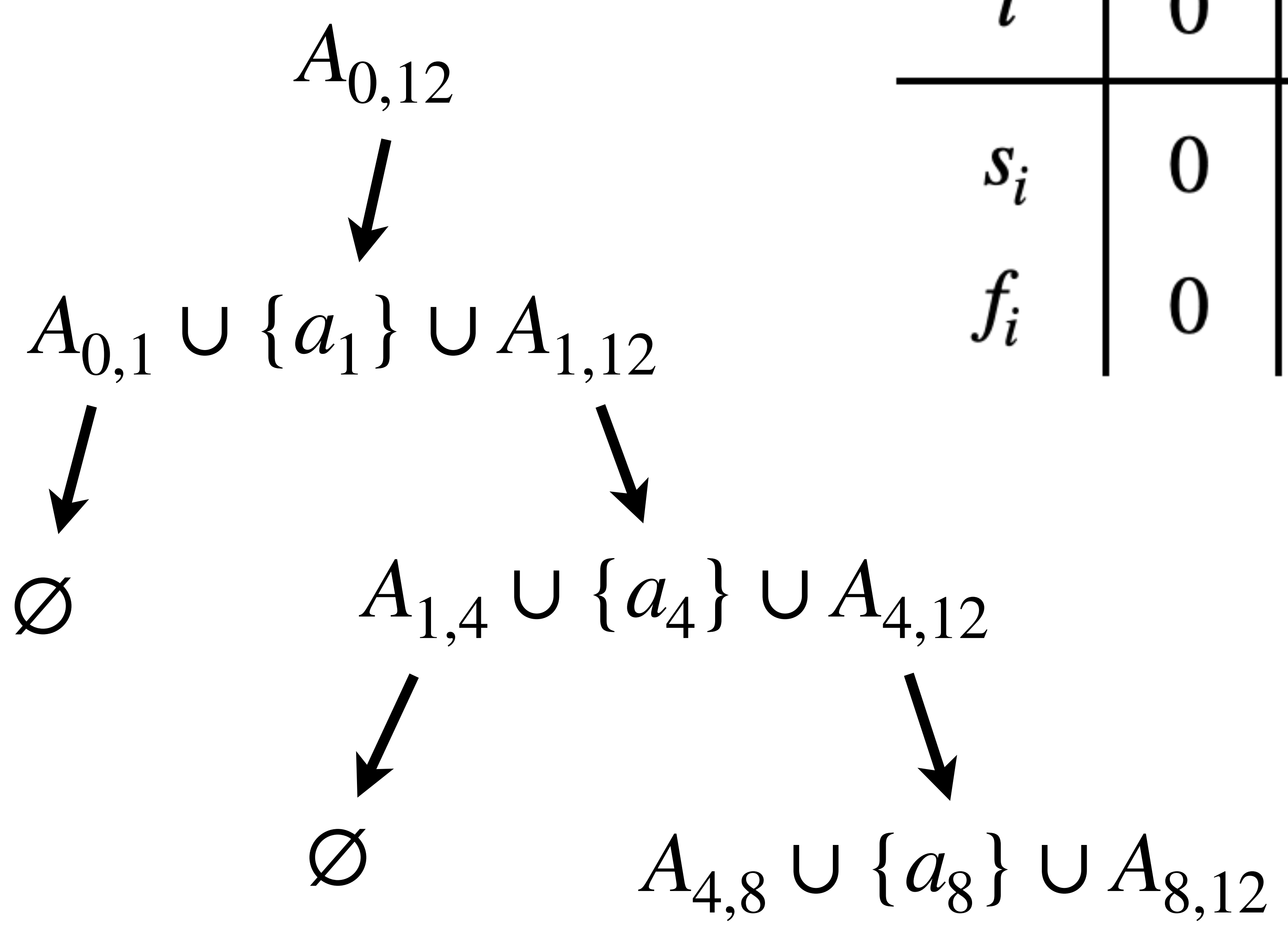

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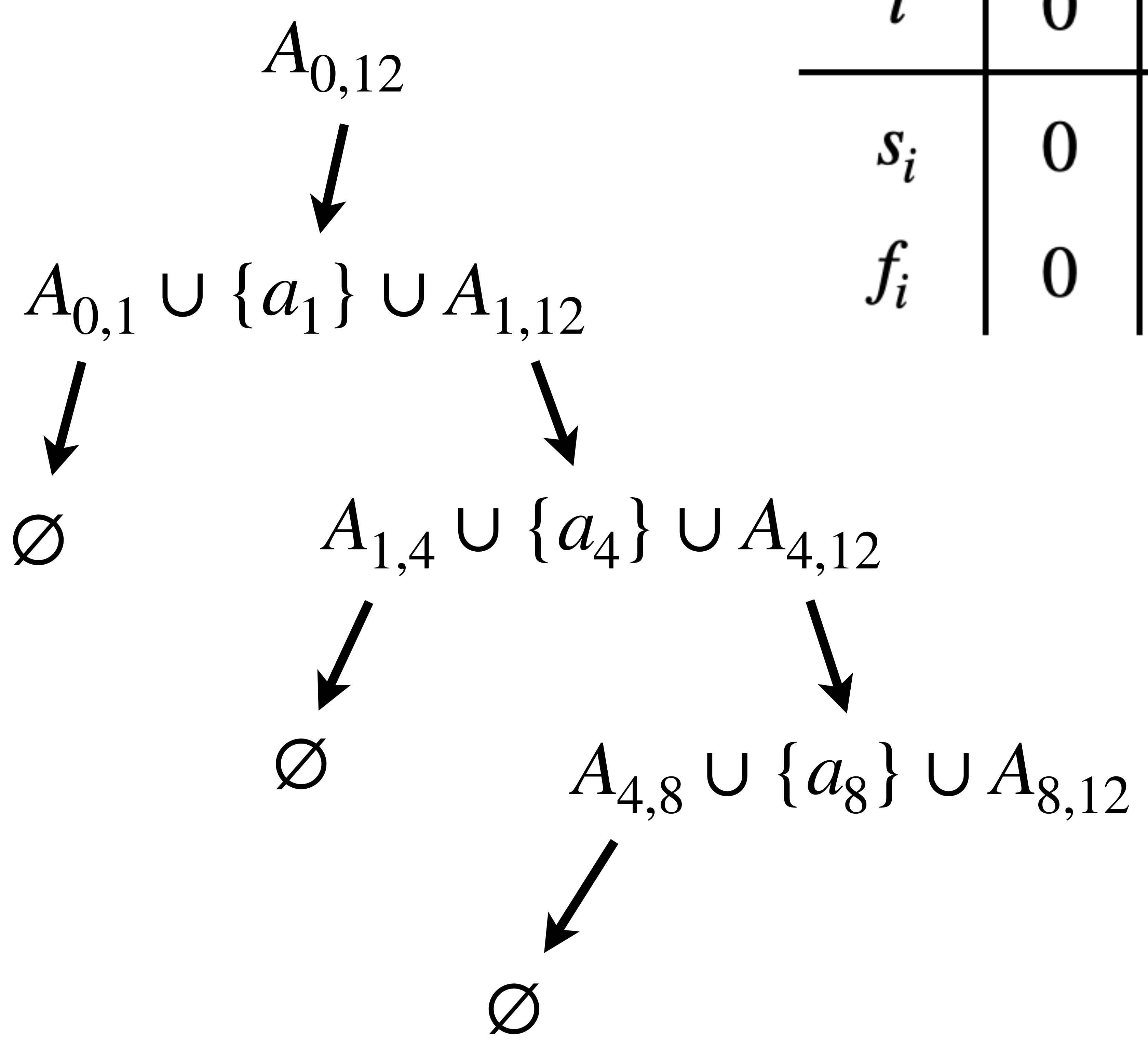

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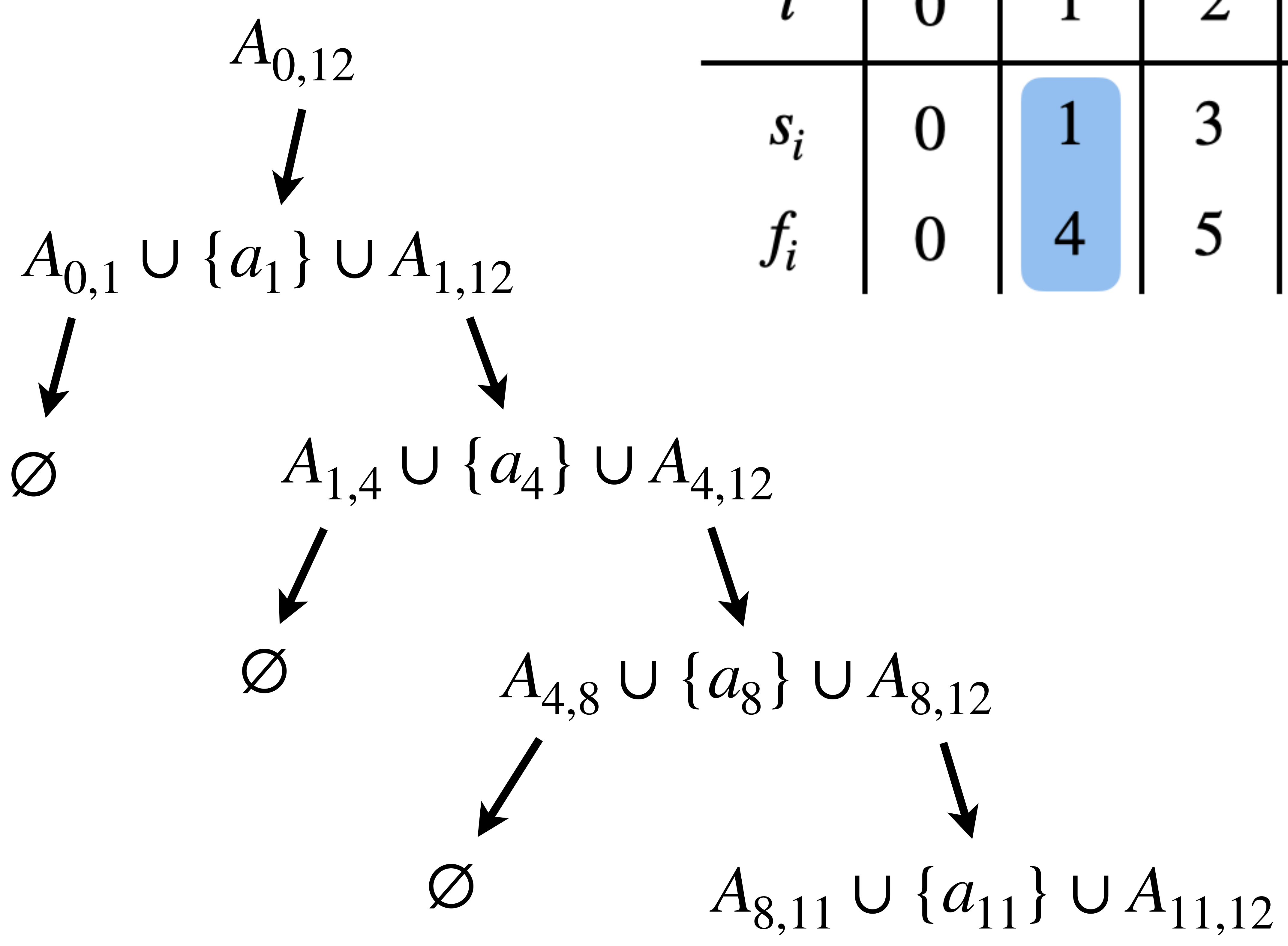

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Greedy Algorithm for Activity-Selection

$A_{0,12}$

$A_{0,1} \cup \{a_1\} \cup A_{1,12}$

$\varnothing$

$A_{1,4} \cup \{a_4\} \cup A_{4,12}$

$\varnothing$

$A_{4,8} \cup \{a_8\} \cup A_{8,12}$

$\varnothing$

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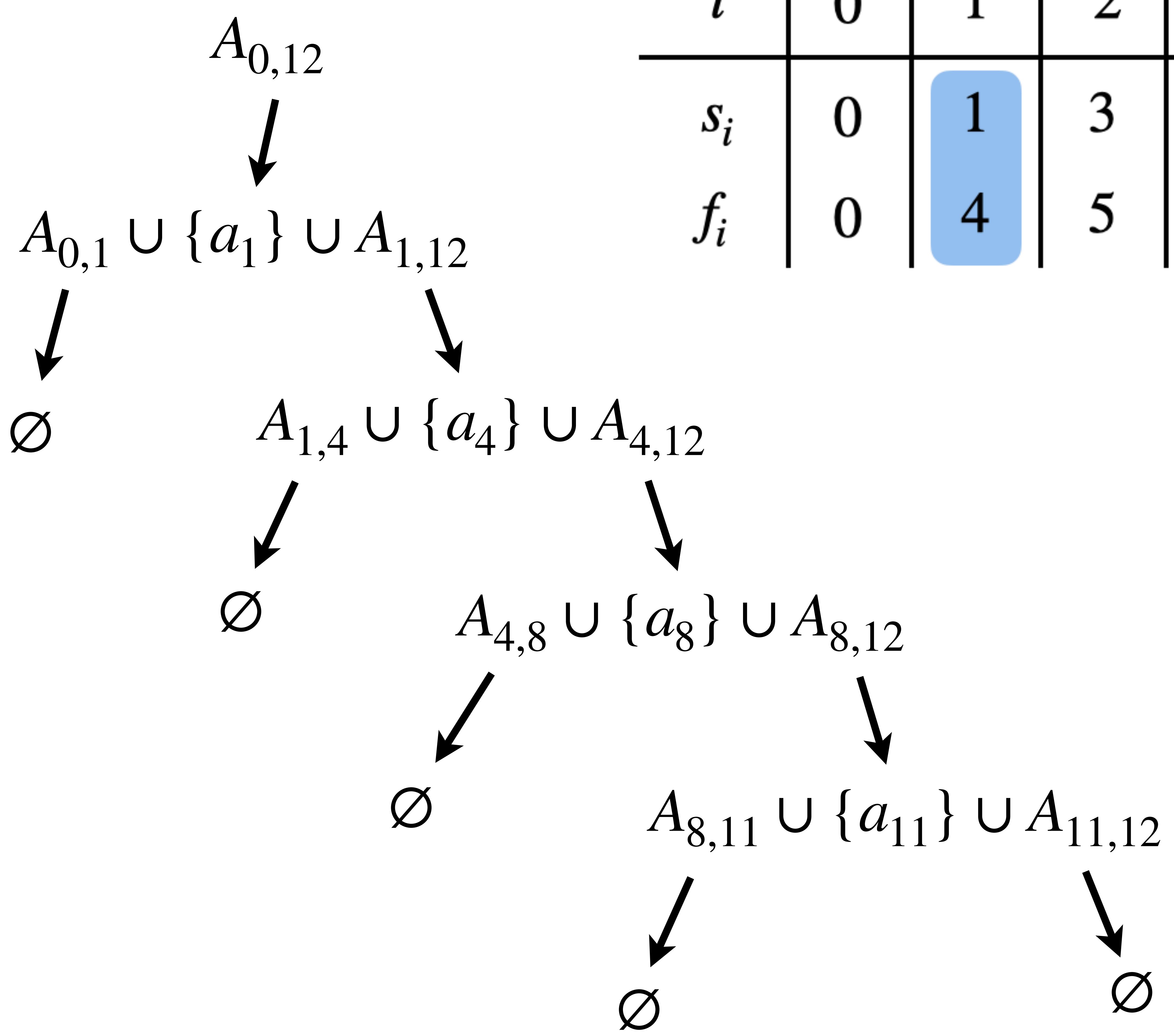

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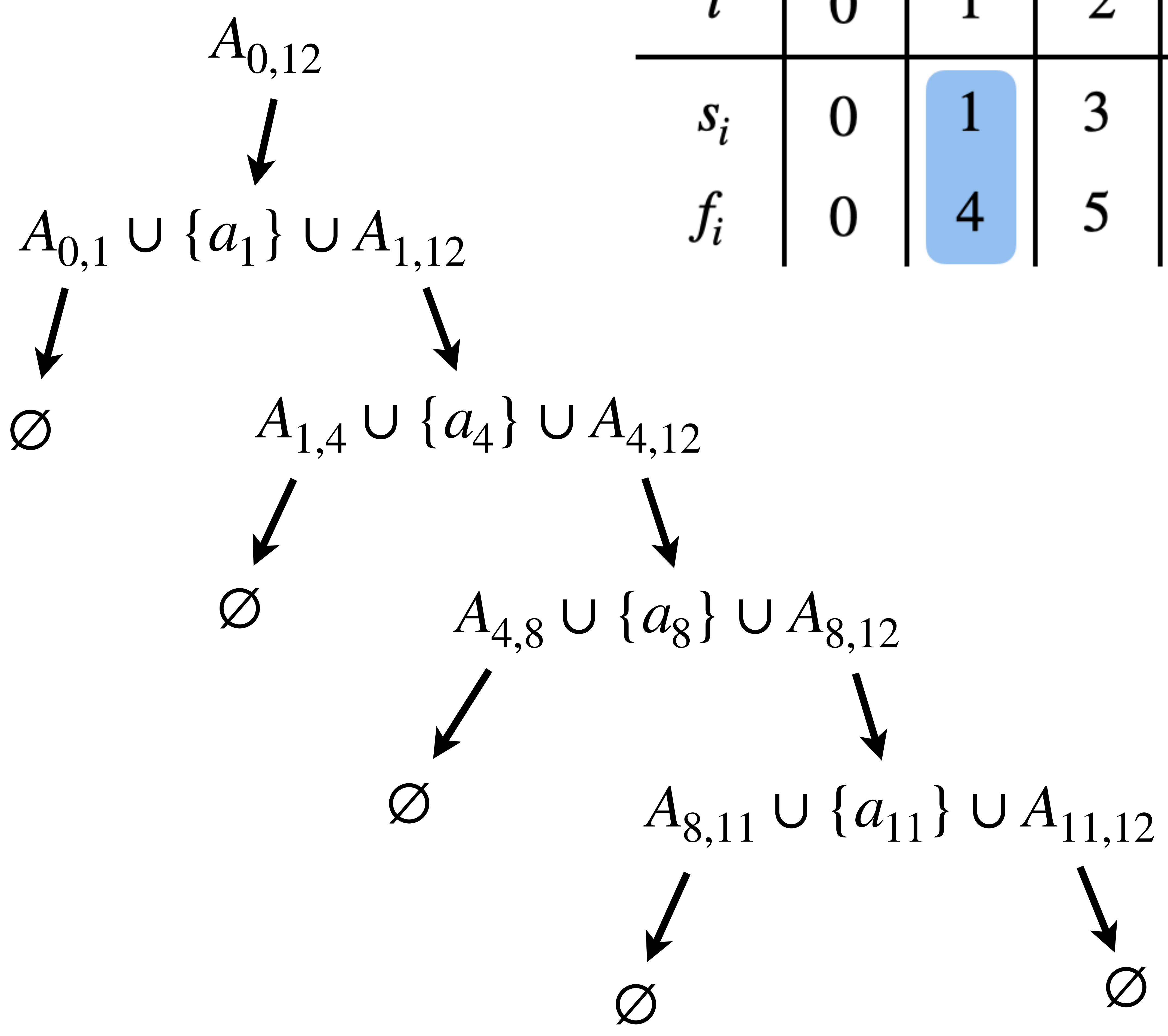
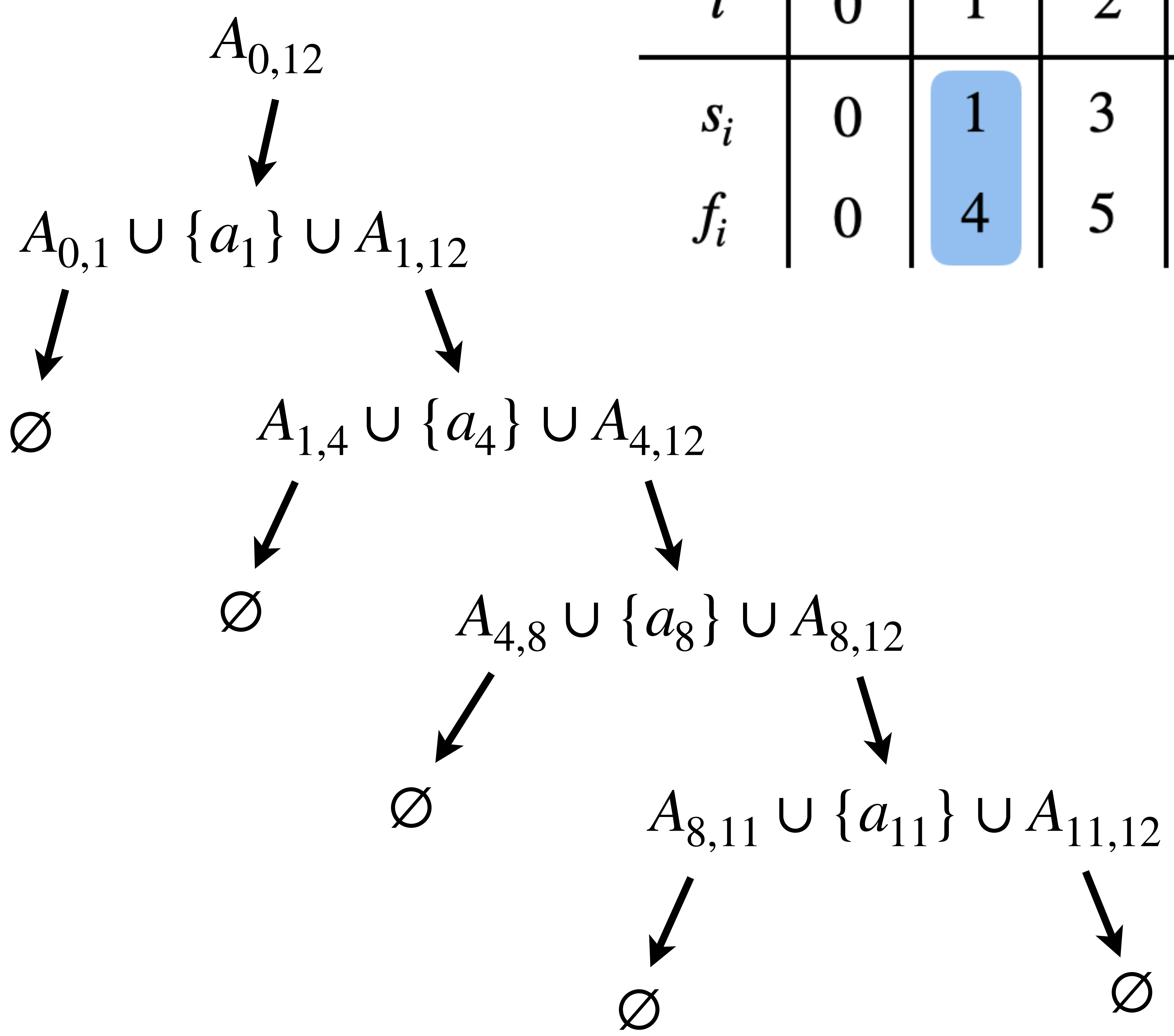


Why $A_{4,8}$ (or $A_{0,1}$, $A_{1,4}$, $A_{8,11}$) must be $\emptyset$?

Greedy Algorithm for Activity-Selection



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If $A_{4,8}$ contains some activity, then we would

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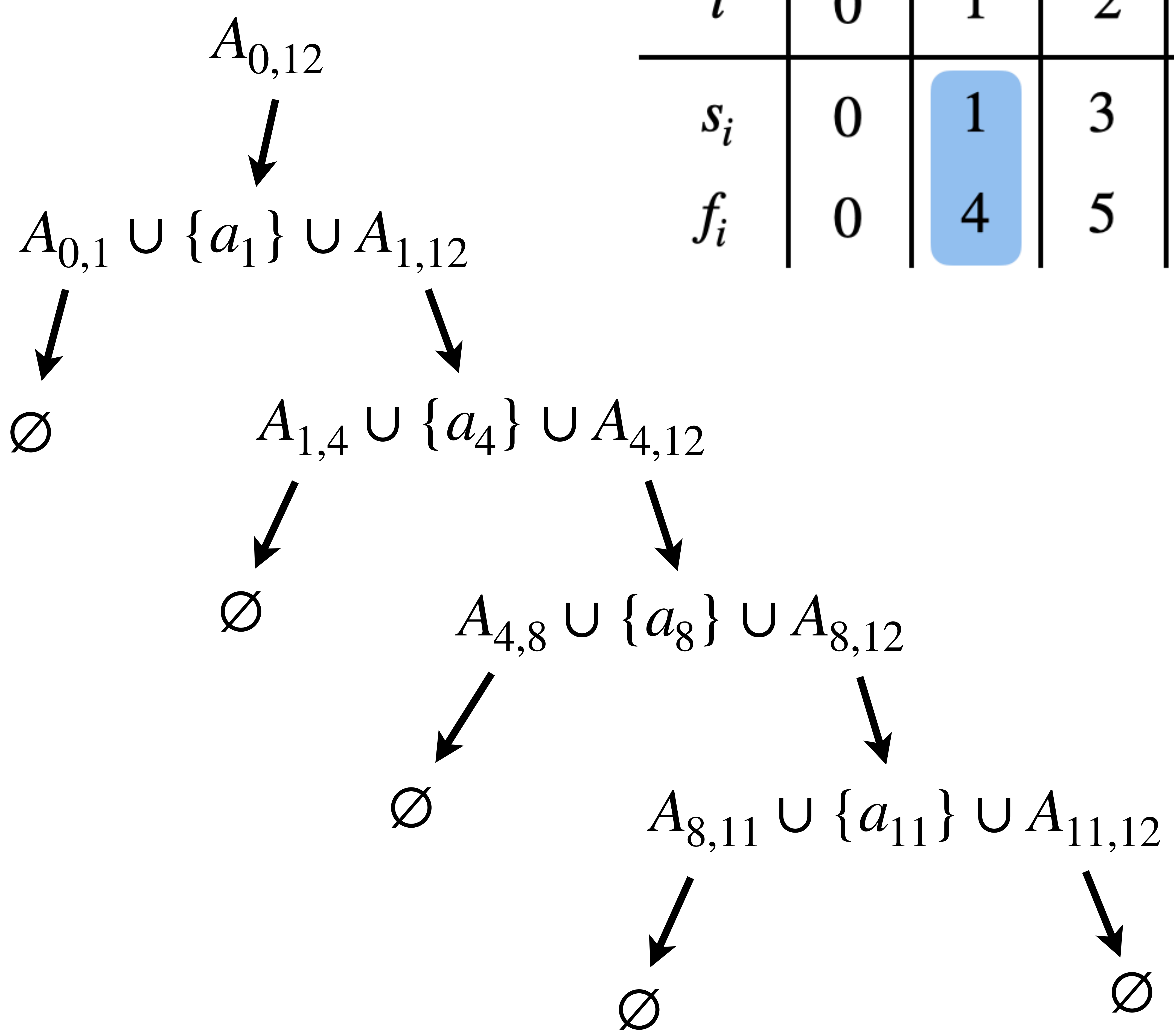

Why $A_{4,8}$ (or $A_{0,1}$, $A_{1,4}$, $A_{8,11}$) must be $\varnothing$?

If $A_{4,8}$ contains some activity, then we would have picked that activity instead of a_8 for $A_{4,12}$.

Greedy Algorithm for Activity-Selection

$$A_{0,12}$$

$$A_{0,1} \cup \{a_1\} \cup A_{1,12}$$

$$\emptyset \qquad A_{1,4} \cup \{a_4\} \cup A_{4,12}$$

$$\emptyset \qquad A_{4,8} \cup \{a_8\} \cup A_{8,12}$$

$$\emptyset \qquad A_{8,11} \cup \{a_{11}\} \cup A_{11,12}$$

$$\emptyset \qquad \emptyset$$

Greedy Algorithm for Activity-Selection

Greedy Strategy:

$$A_{0,12}$$

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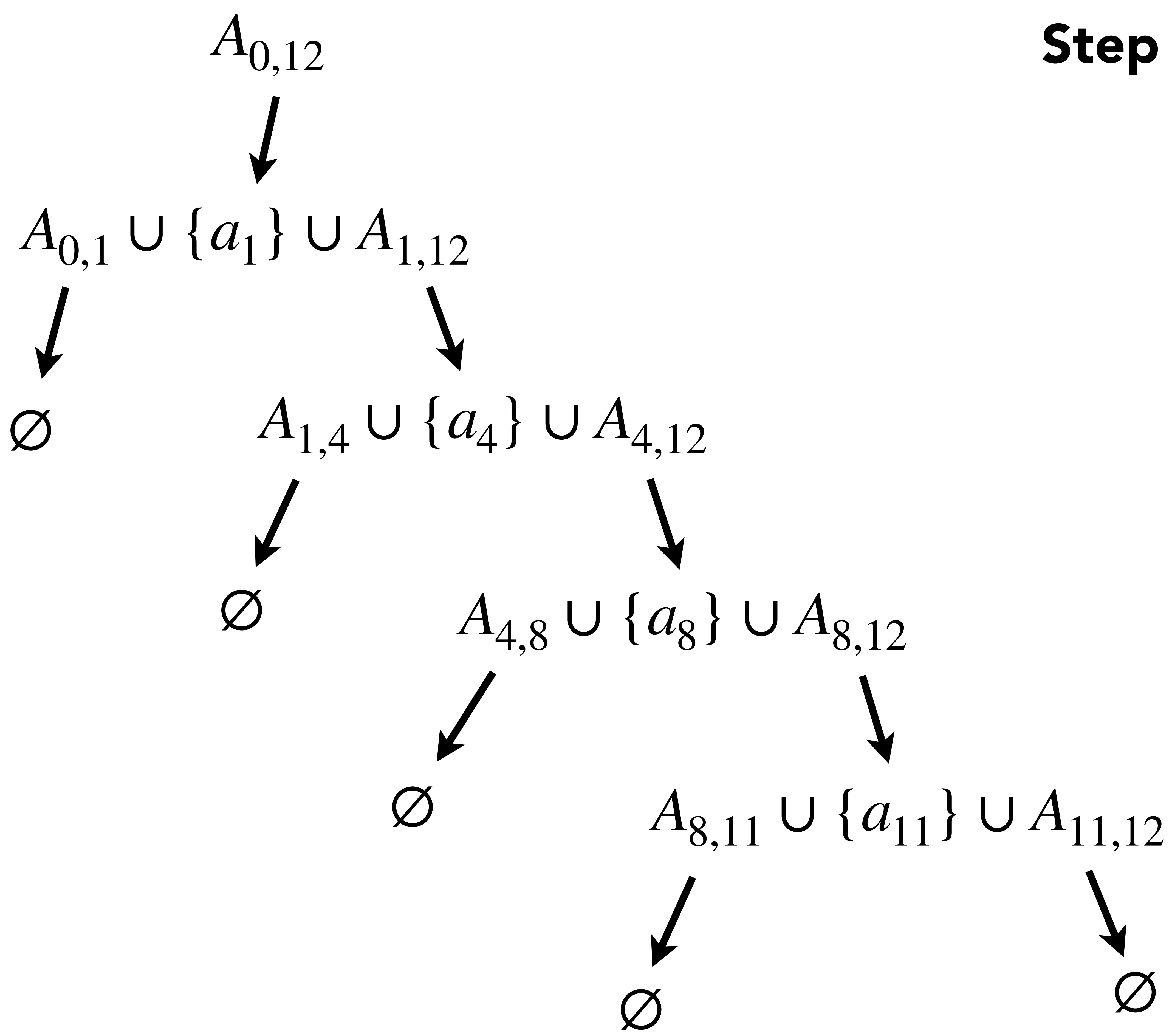
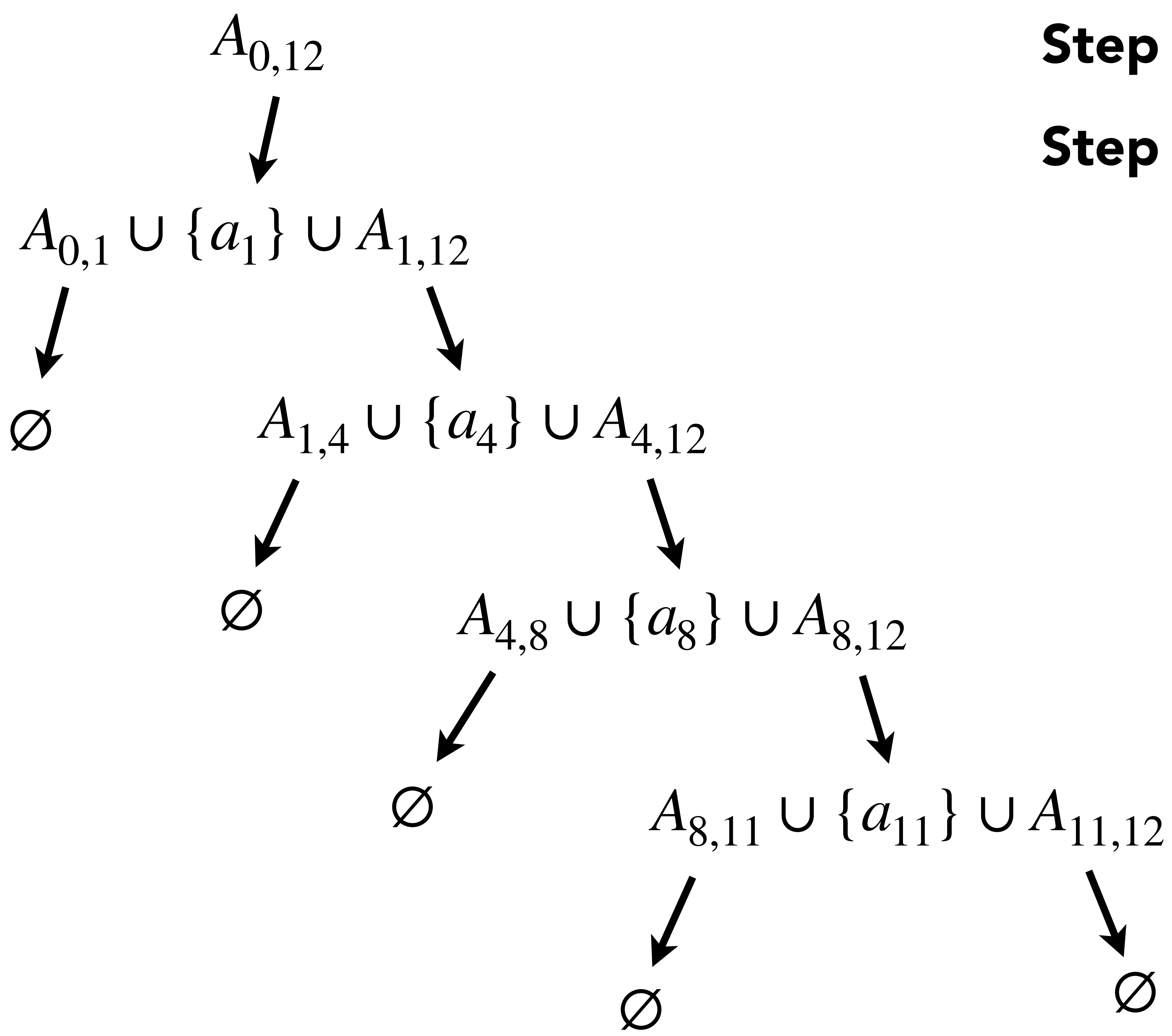
Greedy Algorithm for Activity-Selection



Greedy Strategy:

Step 1: Pick a_1.

Greedy Algorithm for Activity-Selection

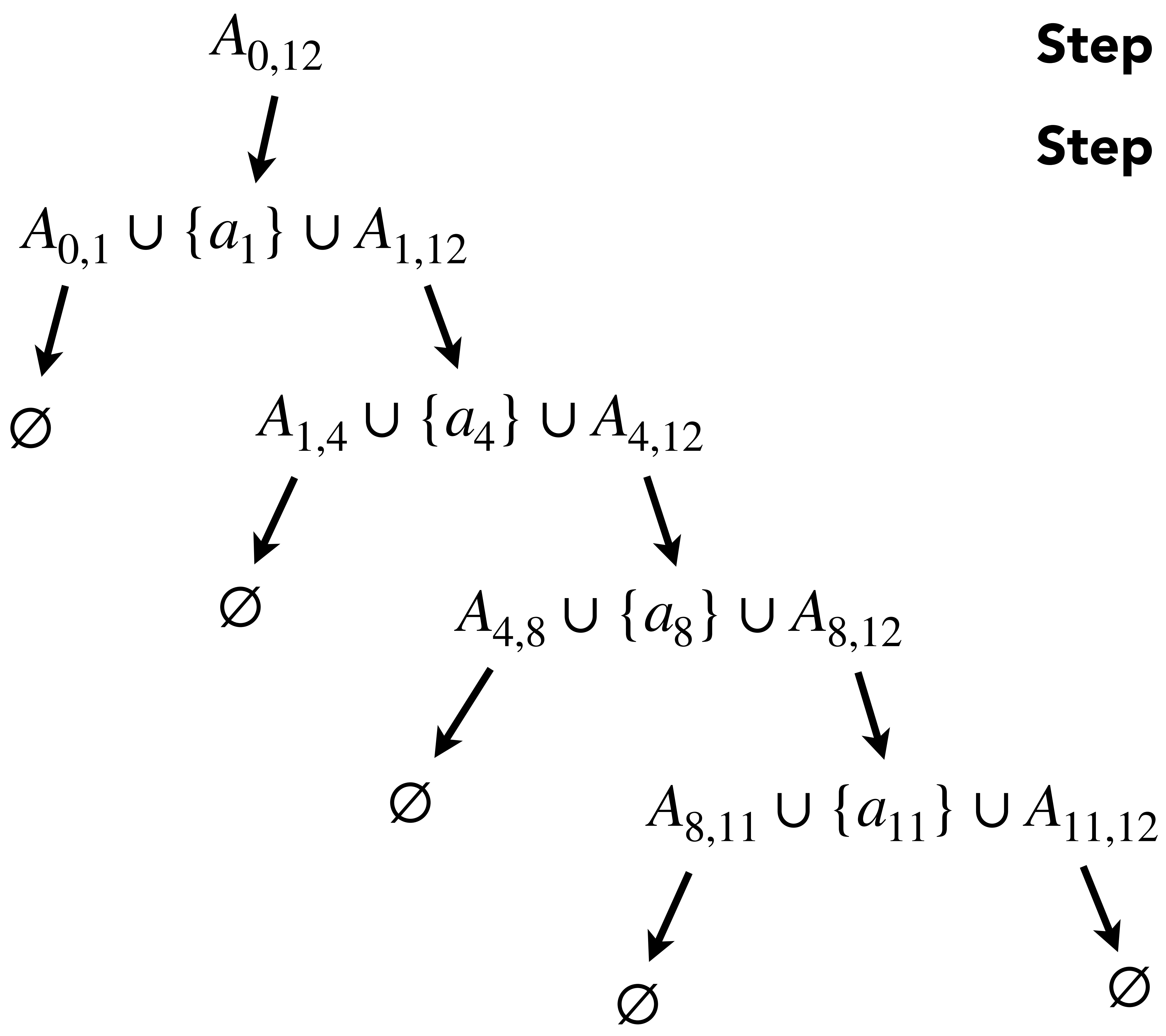


Greedy Strategy:

Step 1: Pick a_1.

Step 2: Let a_k was the last picked activity. Then, pick the

Greedy Algorithm for Activity-Selection



Greedy Strategy:

Step 1: Pick a_1.

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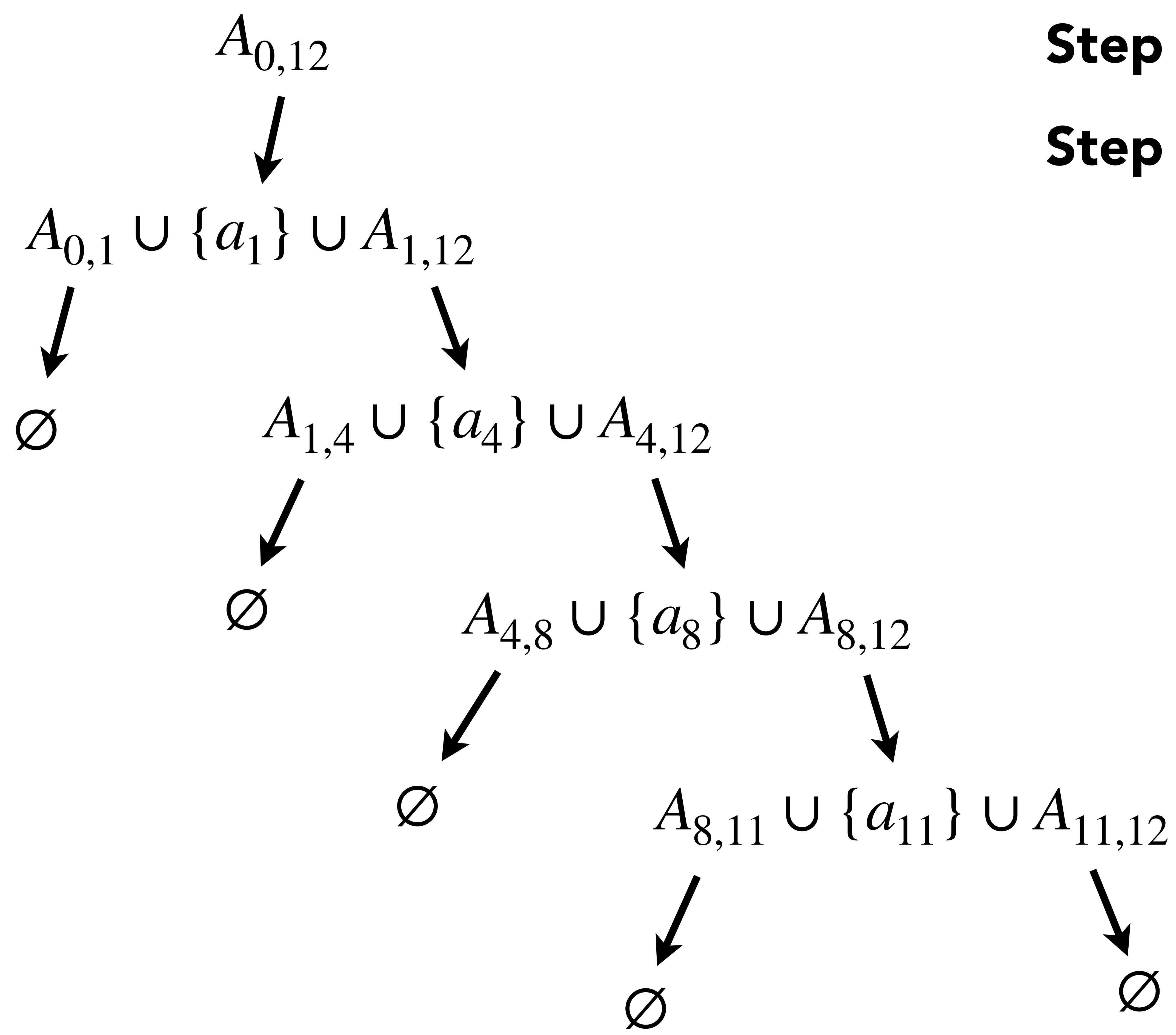
Greedy Algorithm for Activity-Selection



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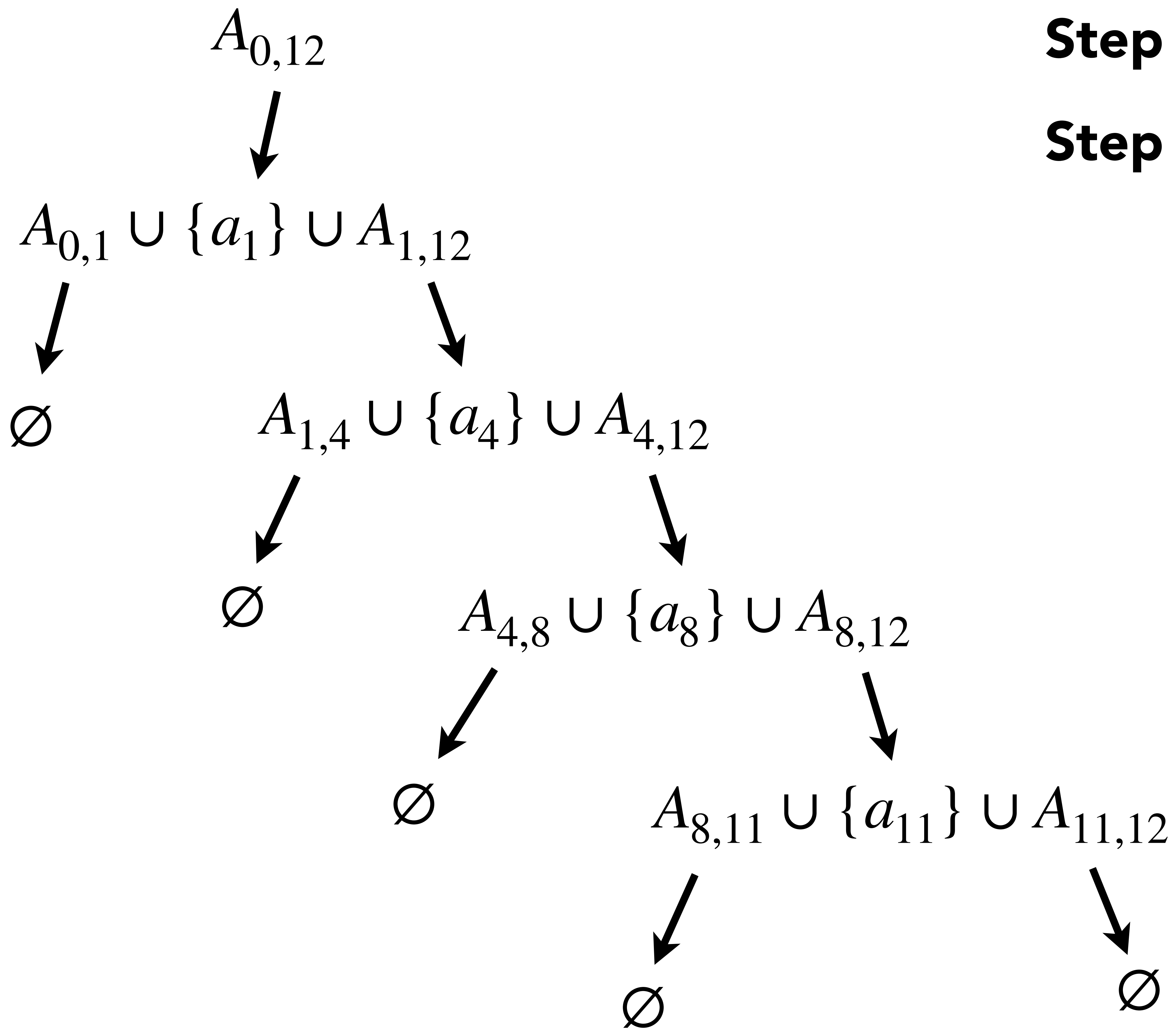

Greedy Strategy:

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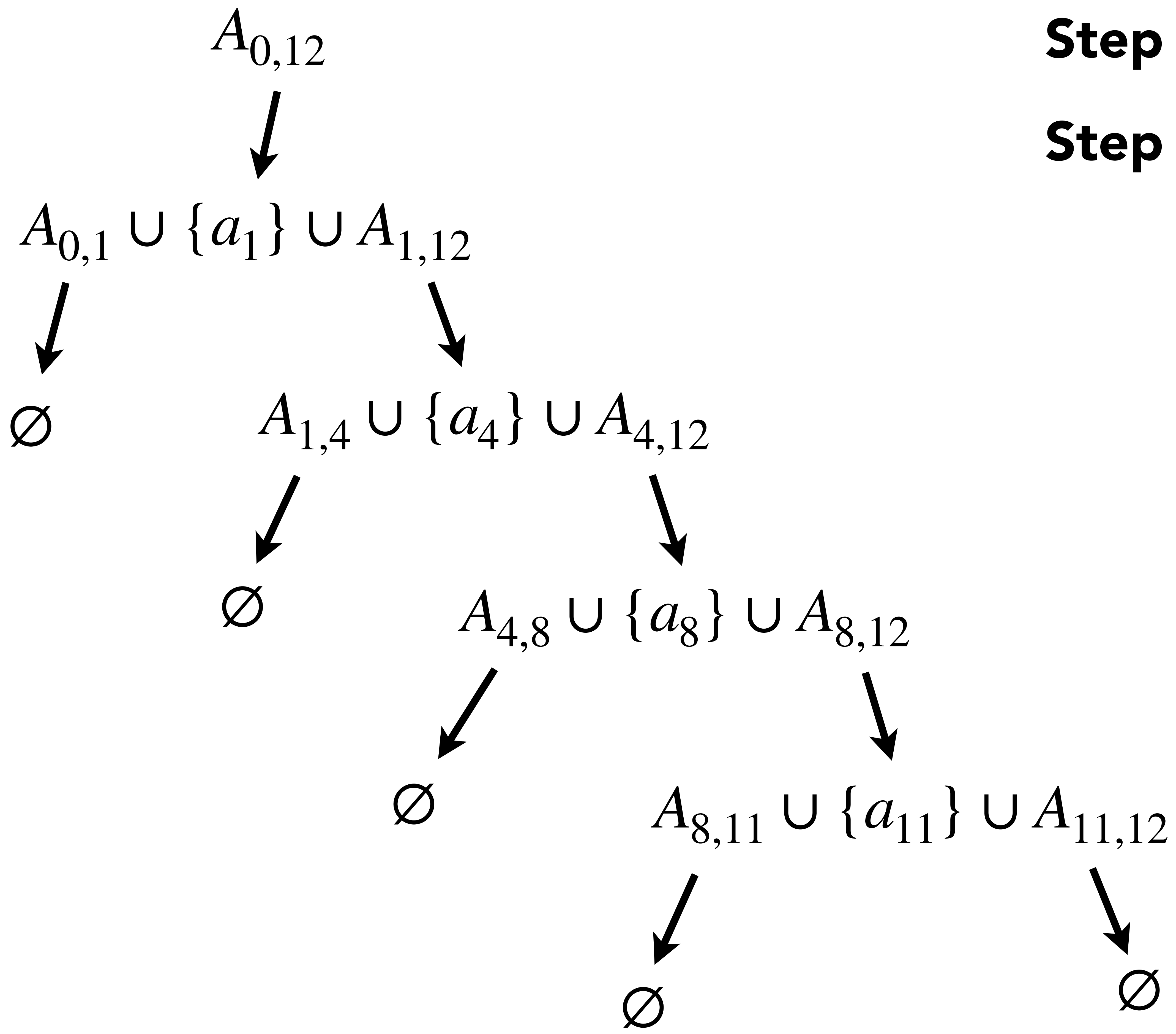

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$$a_k.finish \leq a_i.start \text{ and } a_i.finish \leq a_{n+1}.start$$

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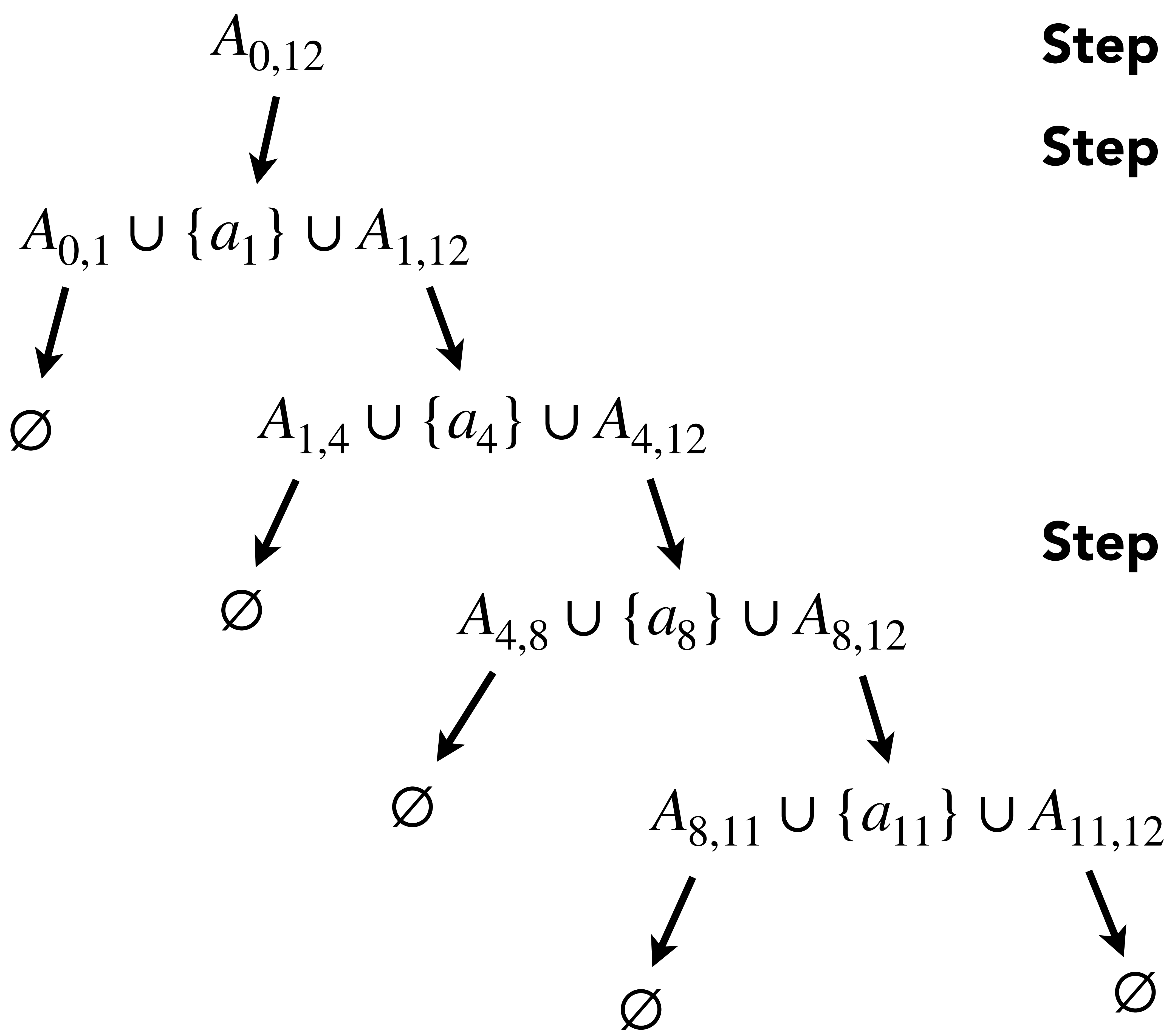

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Step 3**:** Go to **Step** 2, if you can.

Greedy Algorithm for Activity-Selection

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Activity-Selection$(s, f, n+2)$

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Start and finish time of $n+2$ activities (with dummy activities a_0 and a_{n+1})

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1. $A = \{a_1\}$

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Start and finish time of $n + 2$ activities (with dummy activities a_0 and a_{n+1})

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  if $f[k] \leq s[i]$ and $f[i] \leq s[n+1]$

    $A = A \cup \{a_i\}$

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    if $f[k] \leq s[i]$ and $f[i] \leq s[n + 1]$
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        $k = i$ *// resetting the index of the last picked activity*

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return A

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7. **return** A

Will always be true. Hence, can be skipped.

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return A

Will always be true. Hence, can be skipped.

Time Complexity: $\Theta(n)$

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Greedy Choice Property: A globally optimal solution can be constructed by making locally optimal (greedy) choices.

Earliest finishing activity in $S_{i,j}$ will be part of some $A_{i,j}$.

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- Develop the algorithm that implements the greedy strategy.

Note: In practice, one can directly present a greedy algorithm, skipping the above steps..

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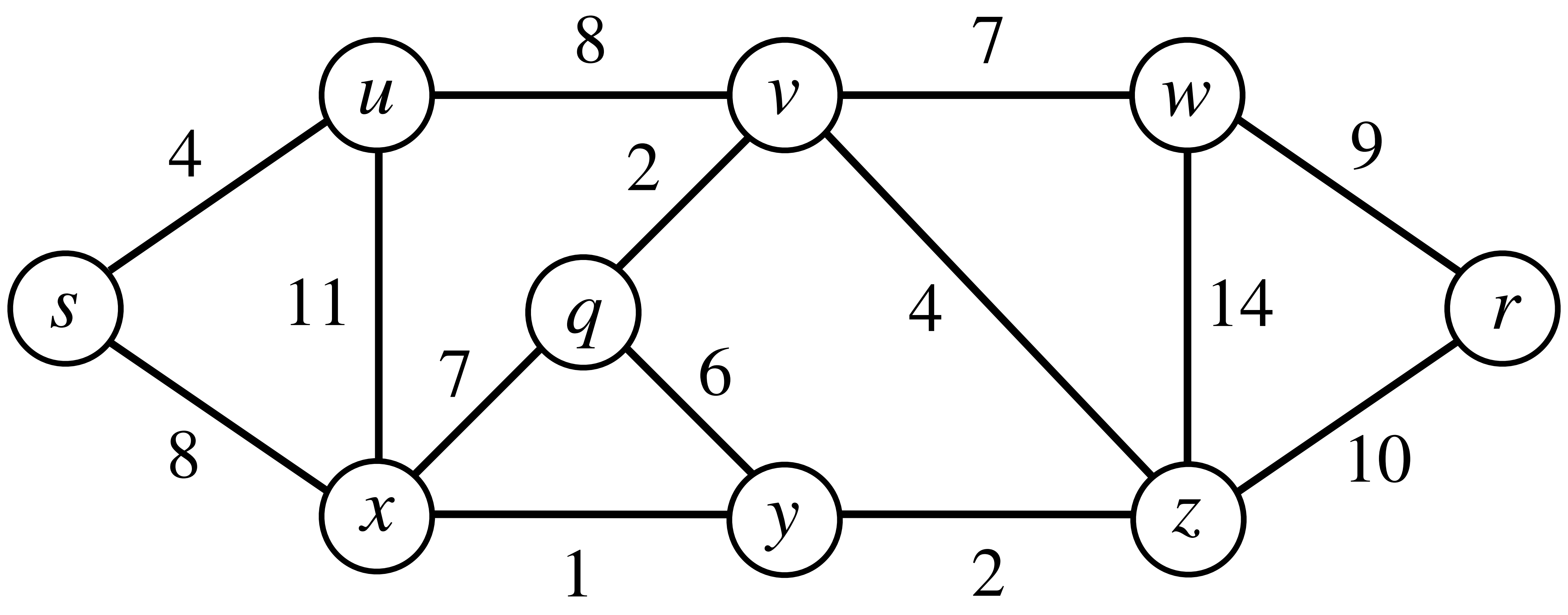

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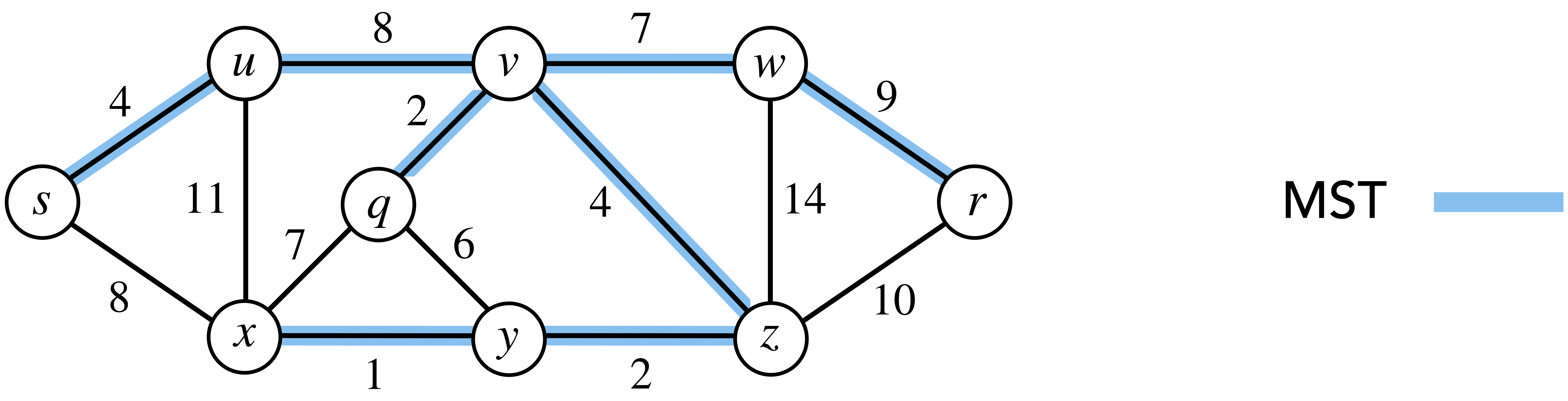

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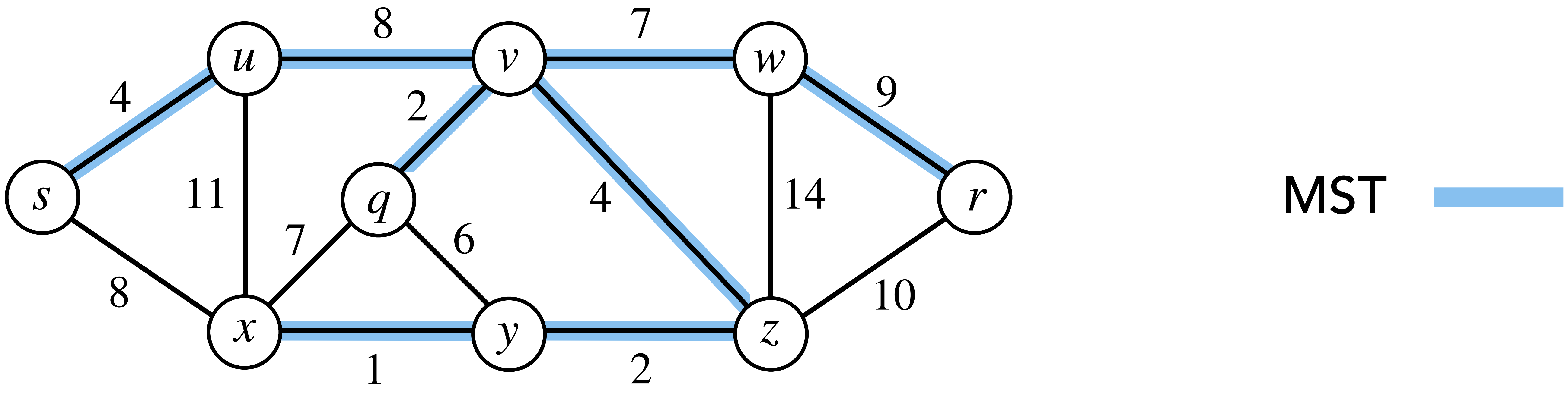

Note: We will represent an MST as a set of edges.

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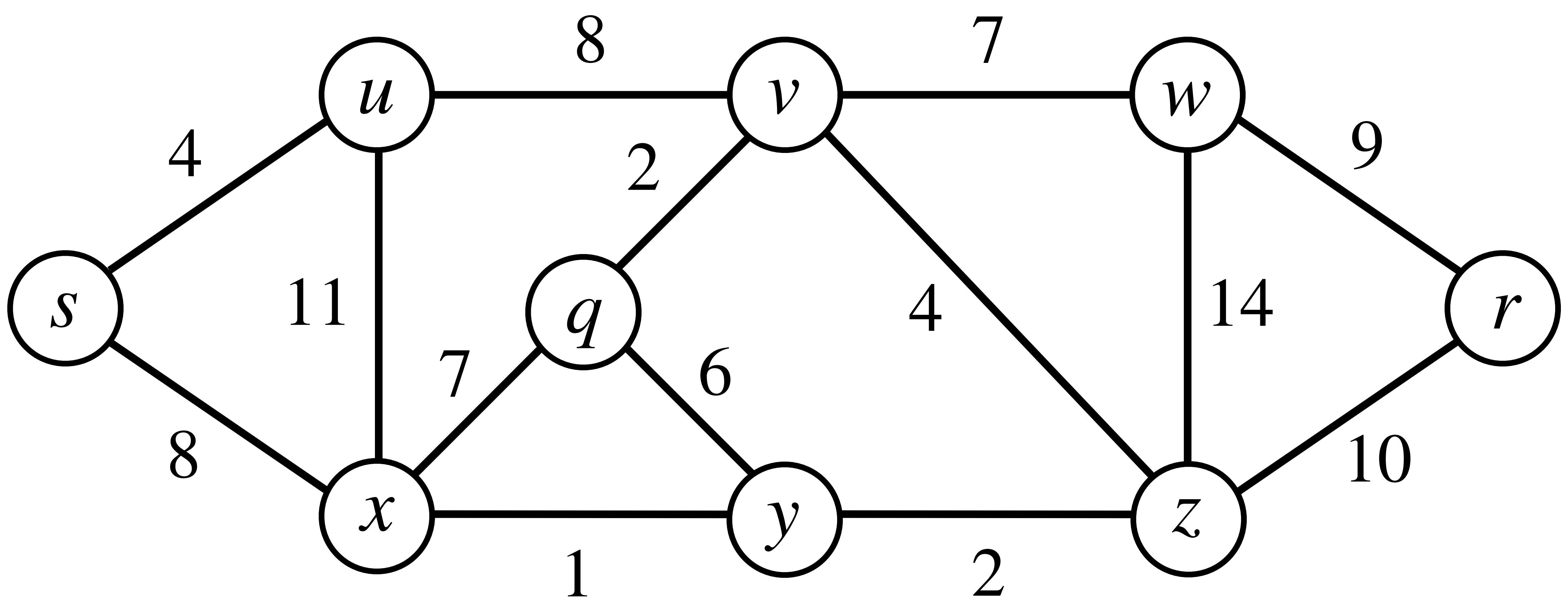

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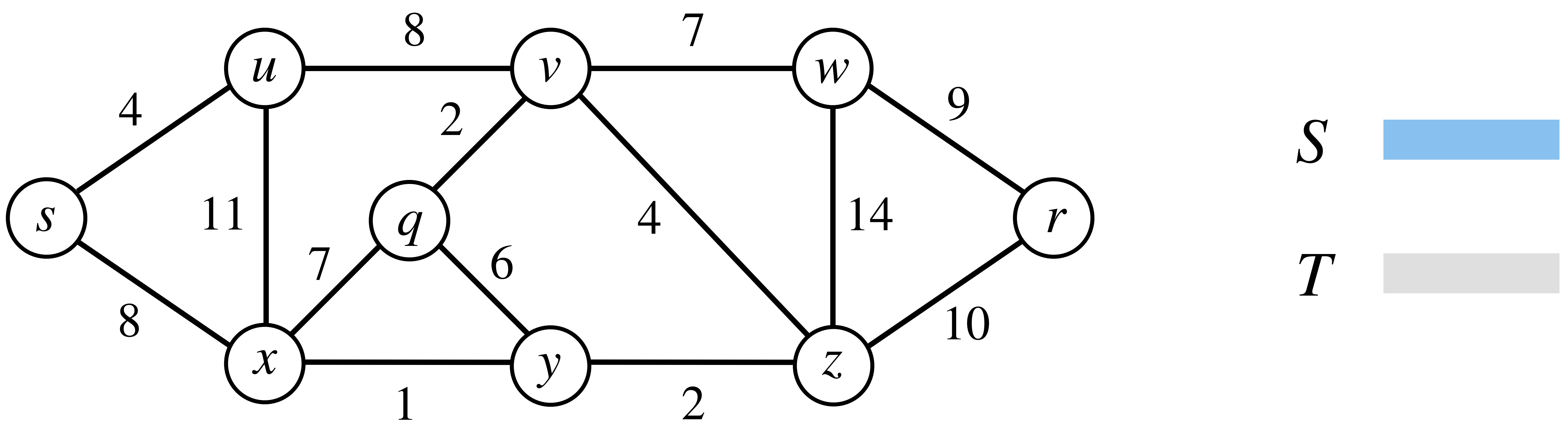

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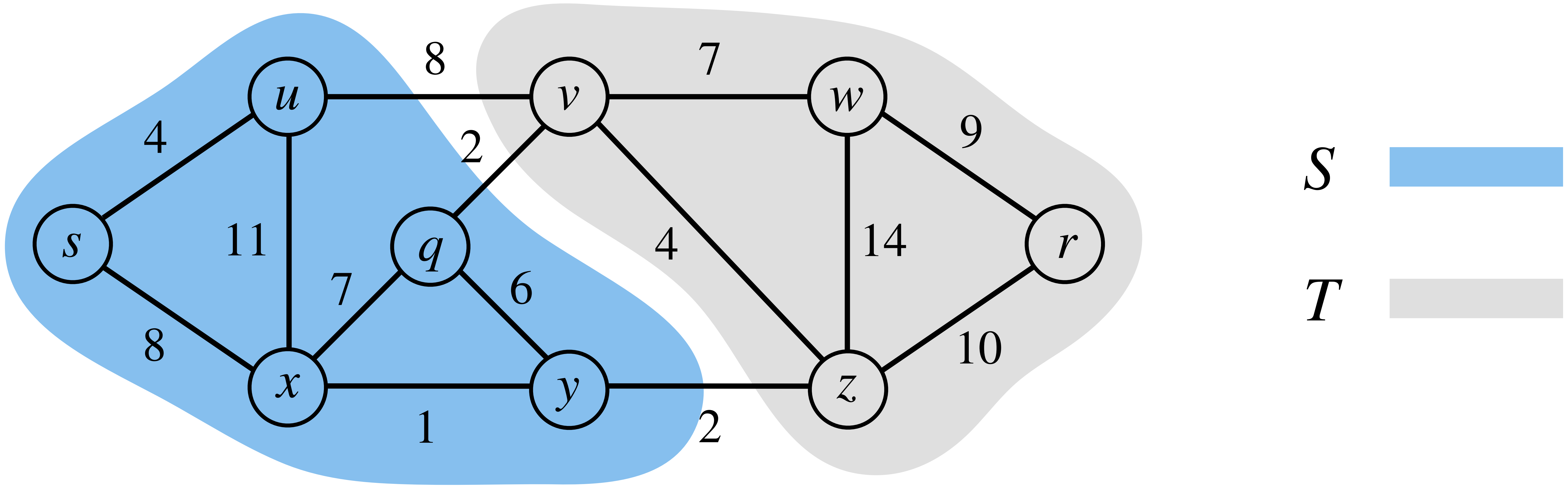
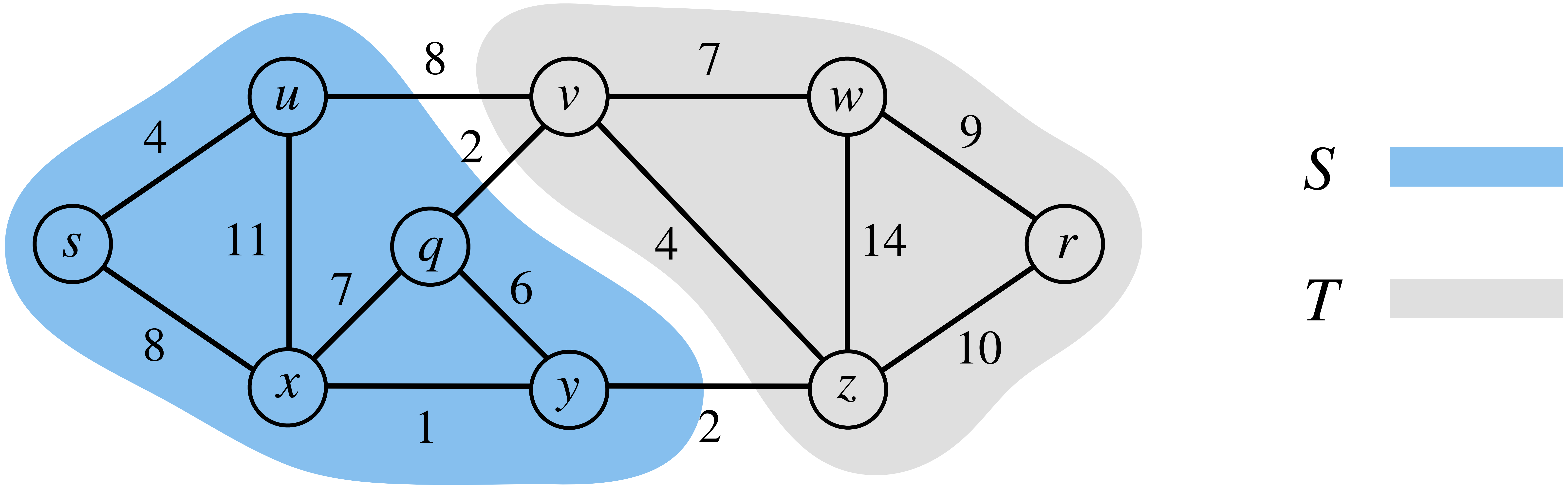


Cut Connection of MST

Defn: A cut $C = (S, T)$ of a graph $G = (V, E)$ is a partition of V in two subsets S and $T = V - S$.

The cut-set of a cut $C = (S, T)$ is the set of edges that have one endpoint in S and other in T.

Example:



The cut-set for cut (S, T) is $\{\{u, v\}, \{q, v\}, \{y, z\}\}$

Cut Connection of MST

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Lemma: Let $C = (S, T)$ be a cut of an undirected, weighted and connected graph $G = (V, E, w)$.

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Proof: On the next slide.

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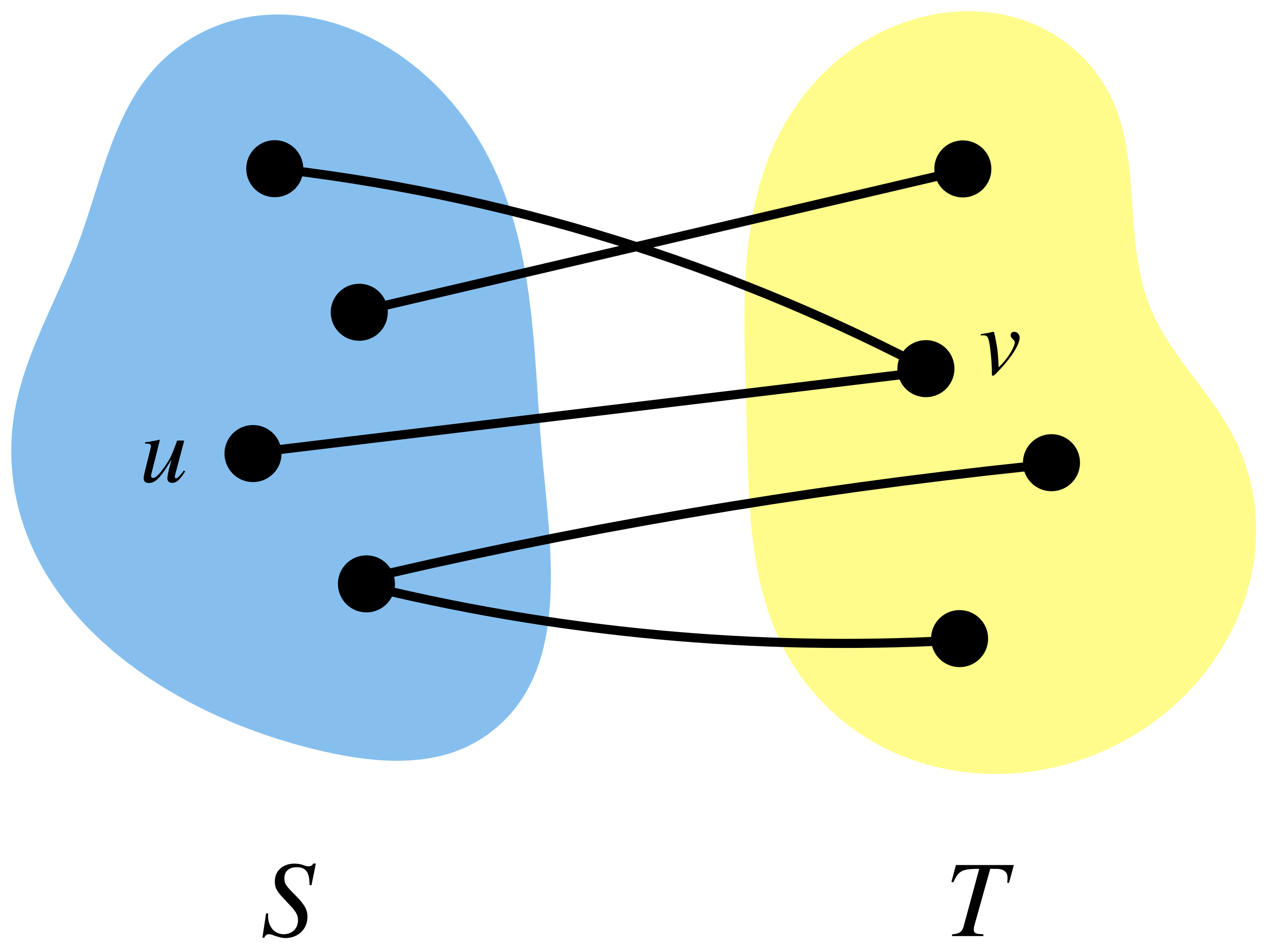
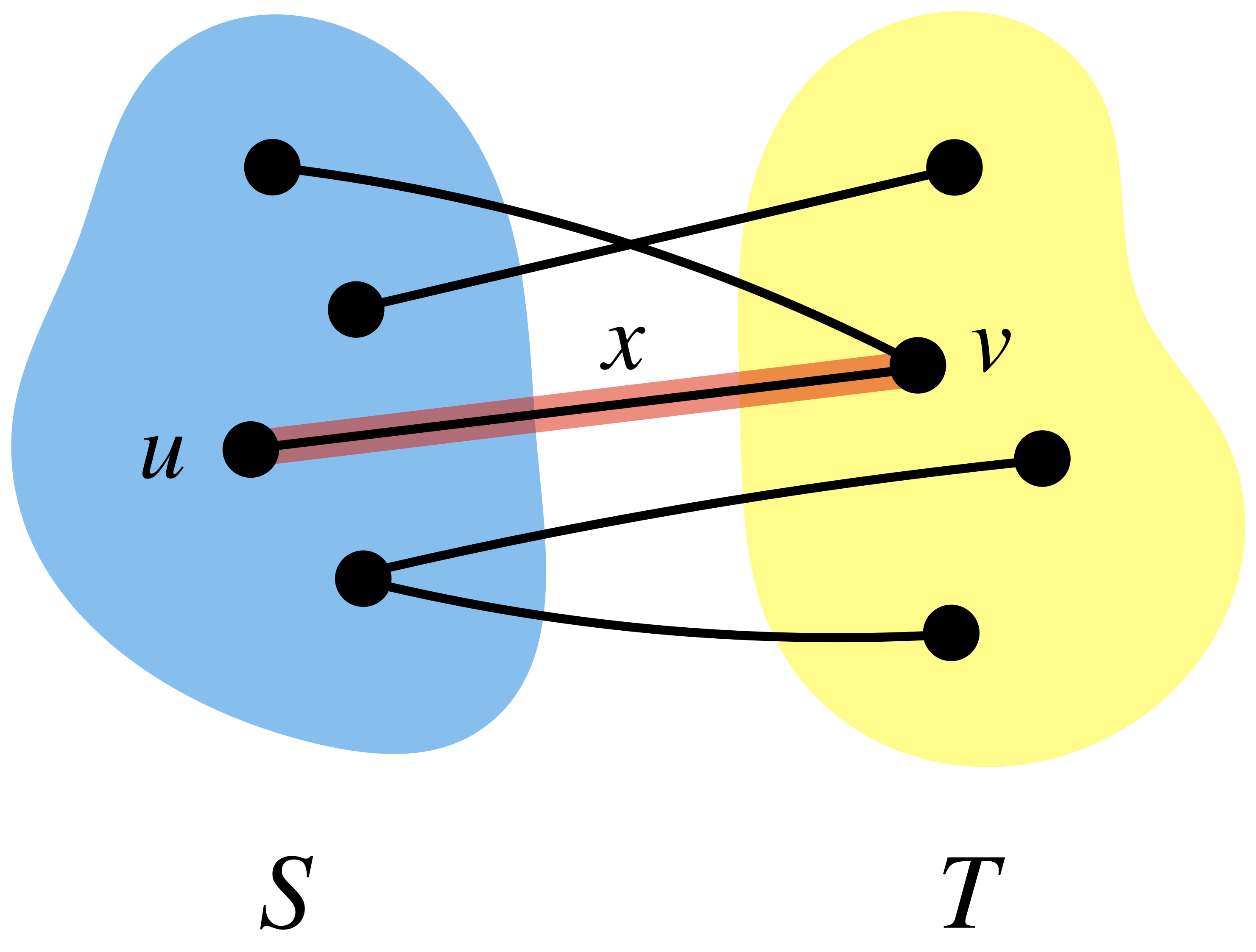


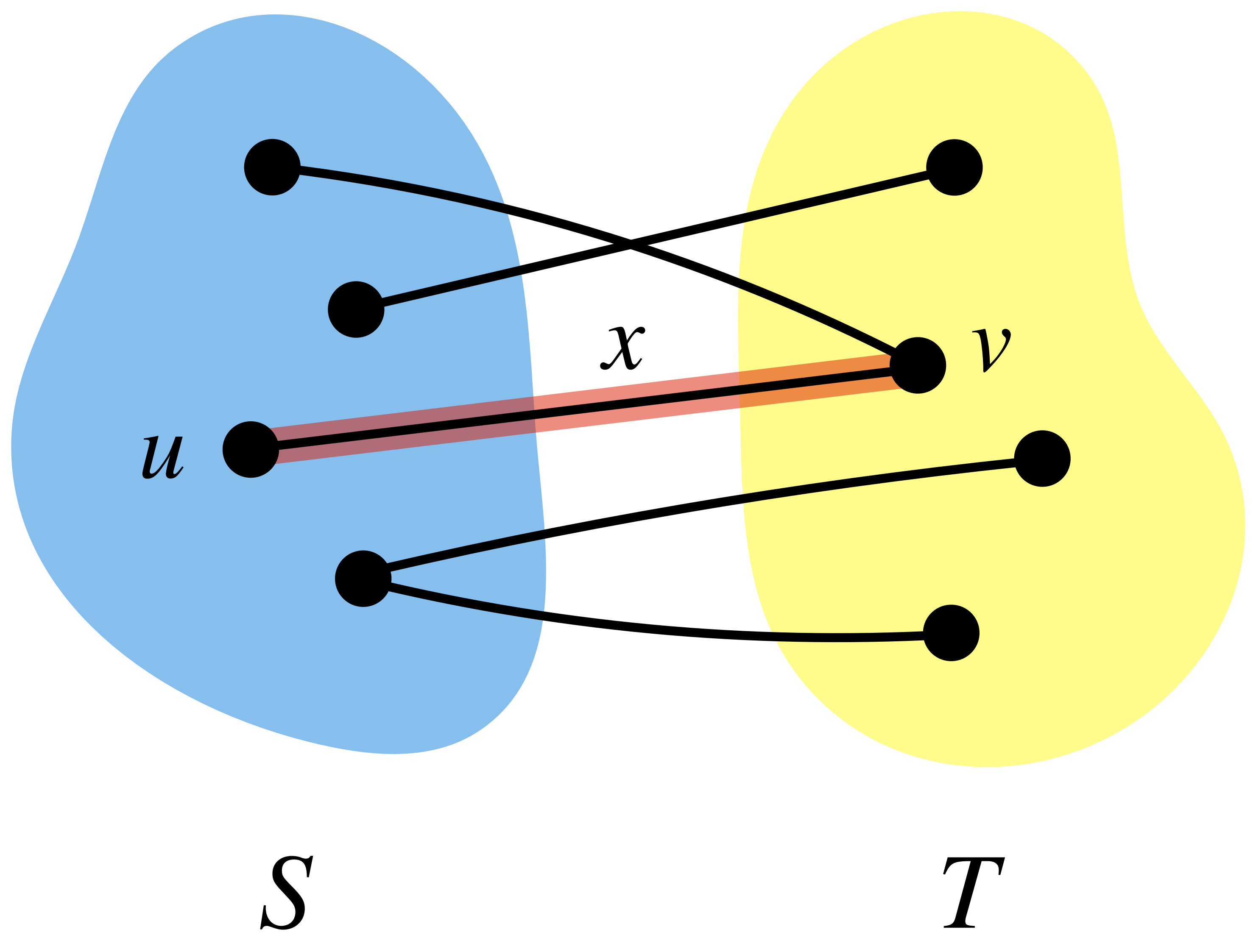
Cut Connection of MST

Proof:



Cut Connection of MST

Proof: Let $\{u, v\}$ be a least weight edge in the cut-set of C with weight x.



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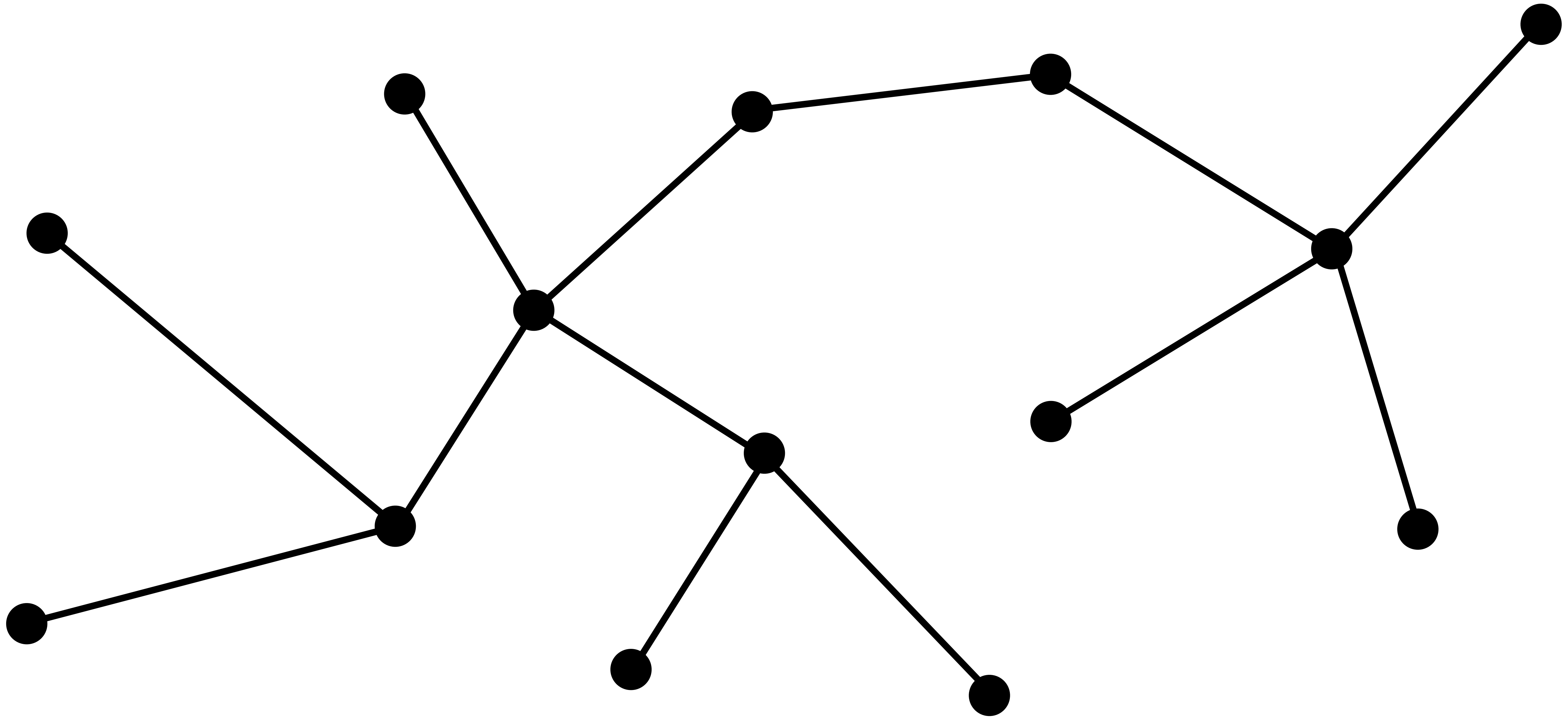

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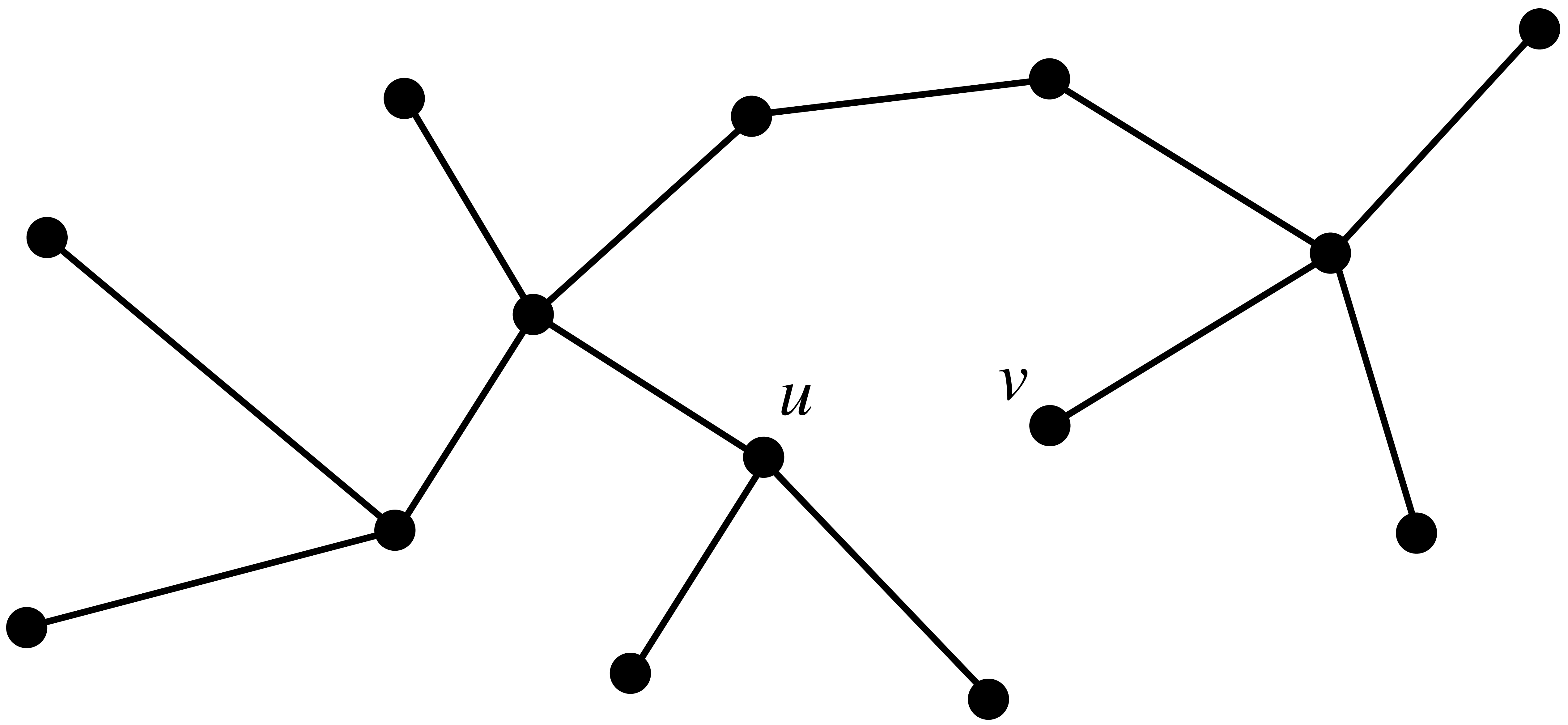

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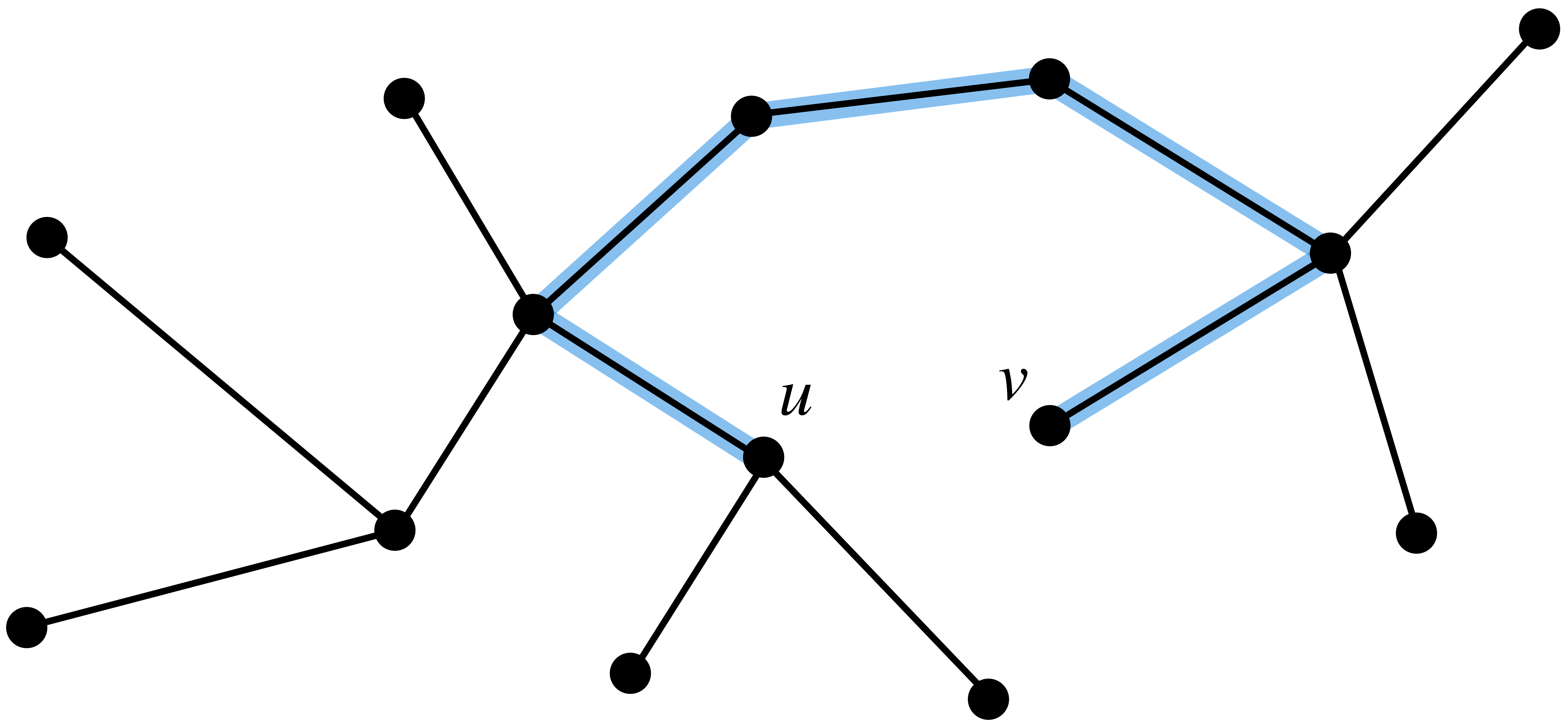

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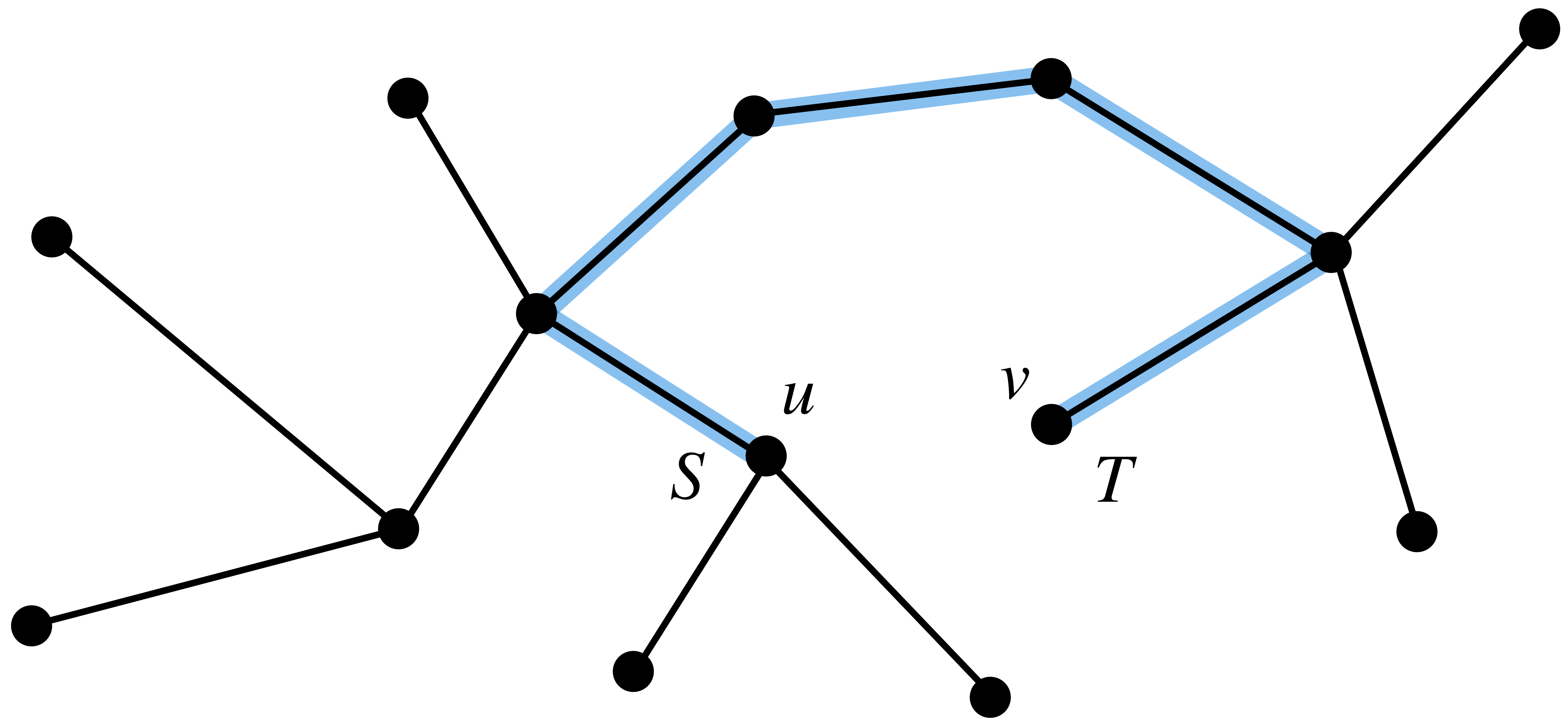

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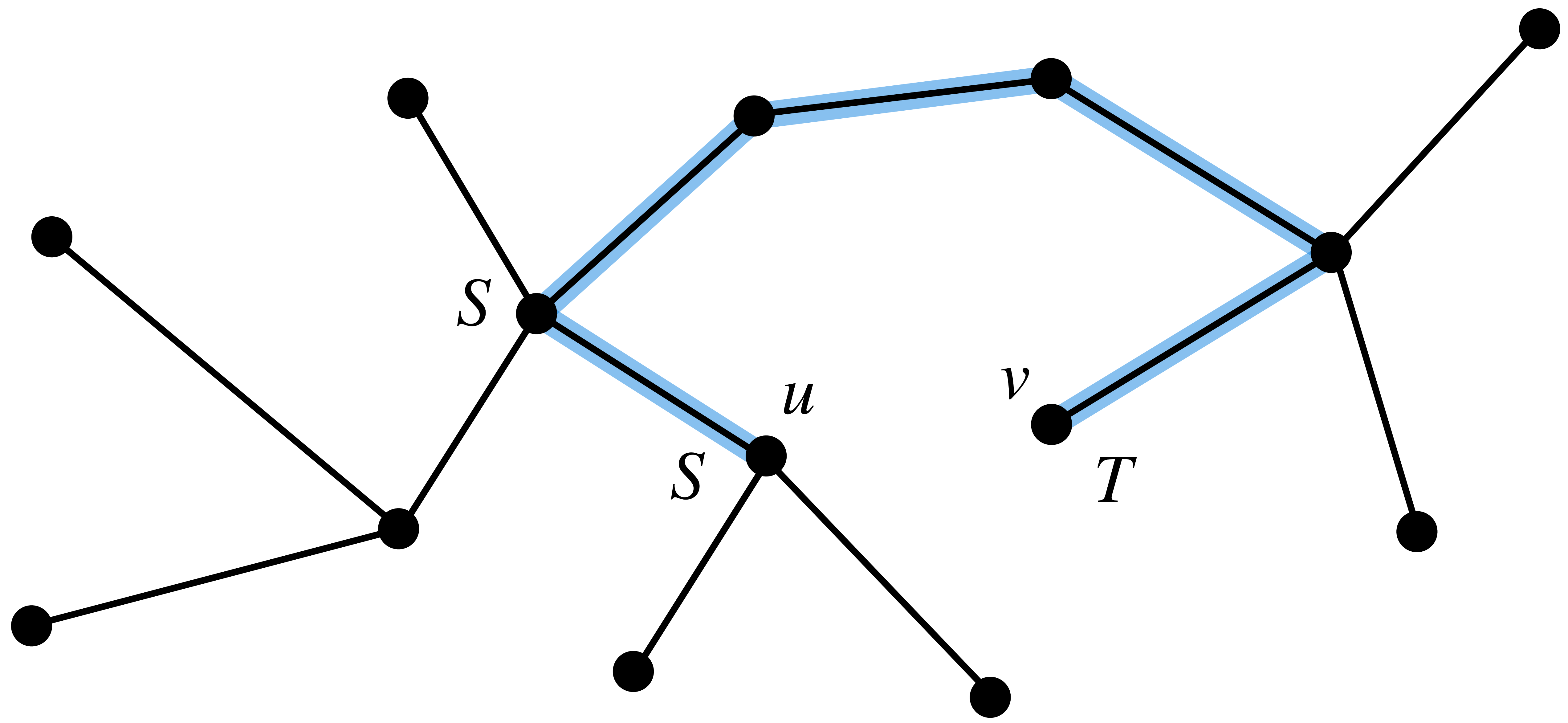

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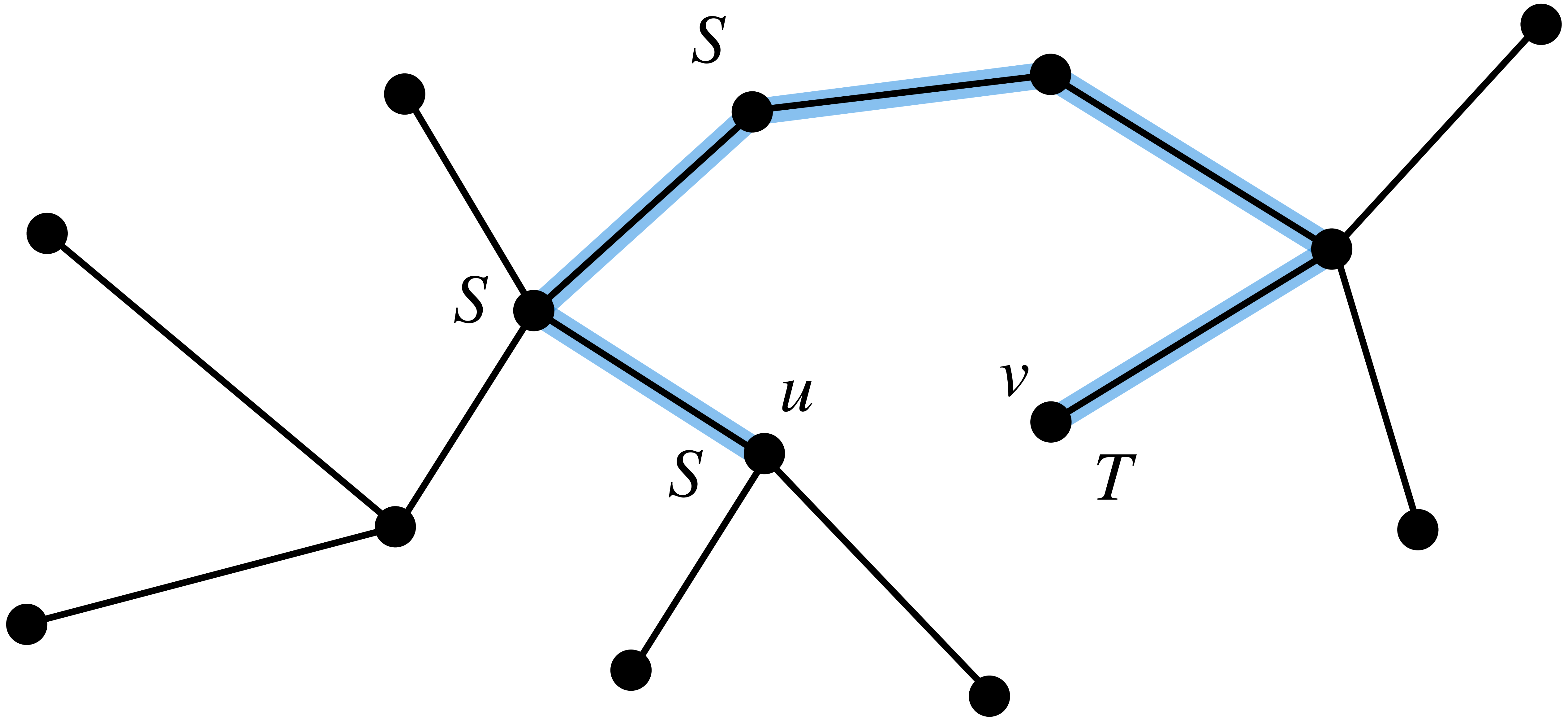

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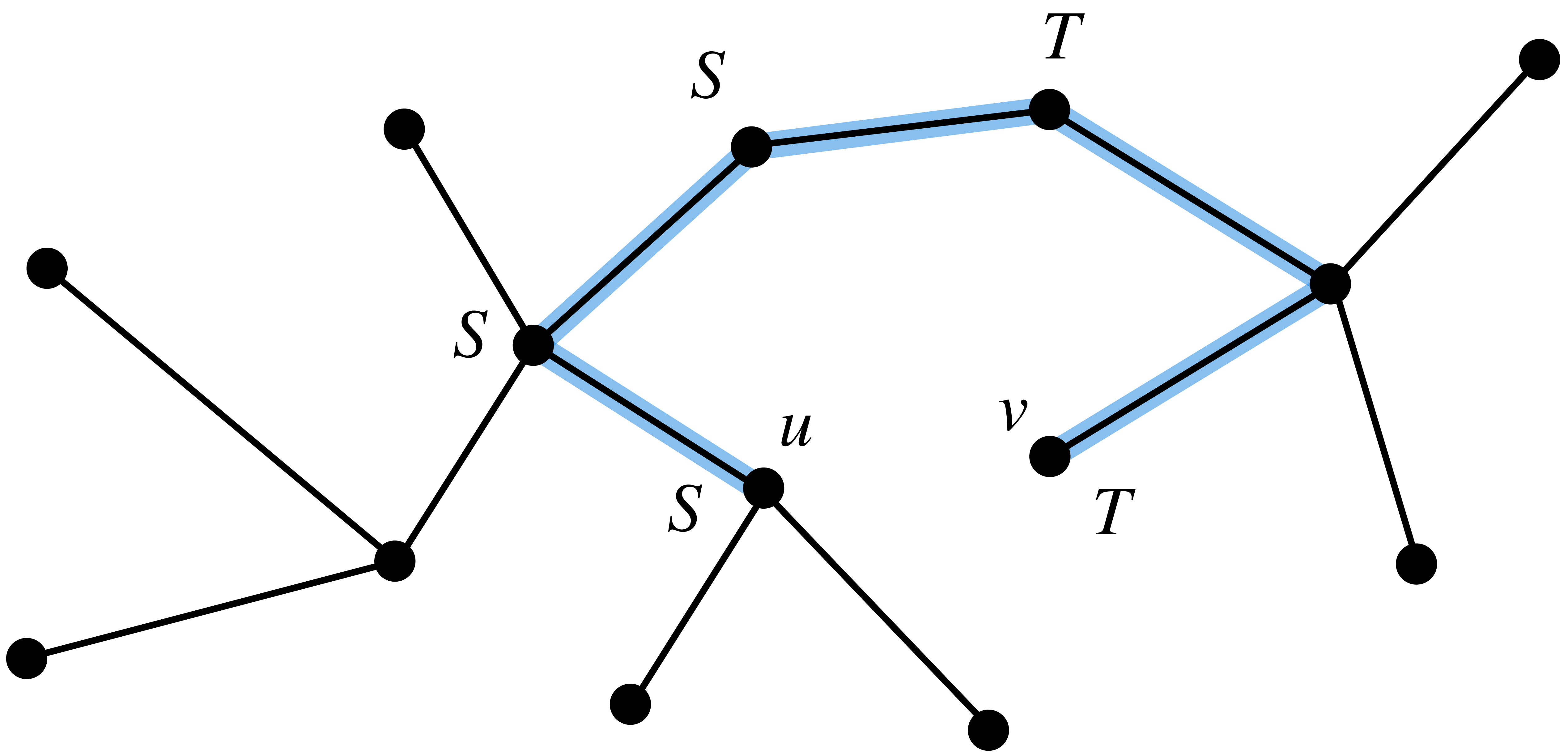

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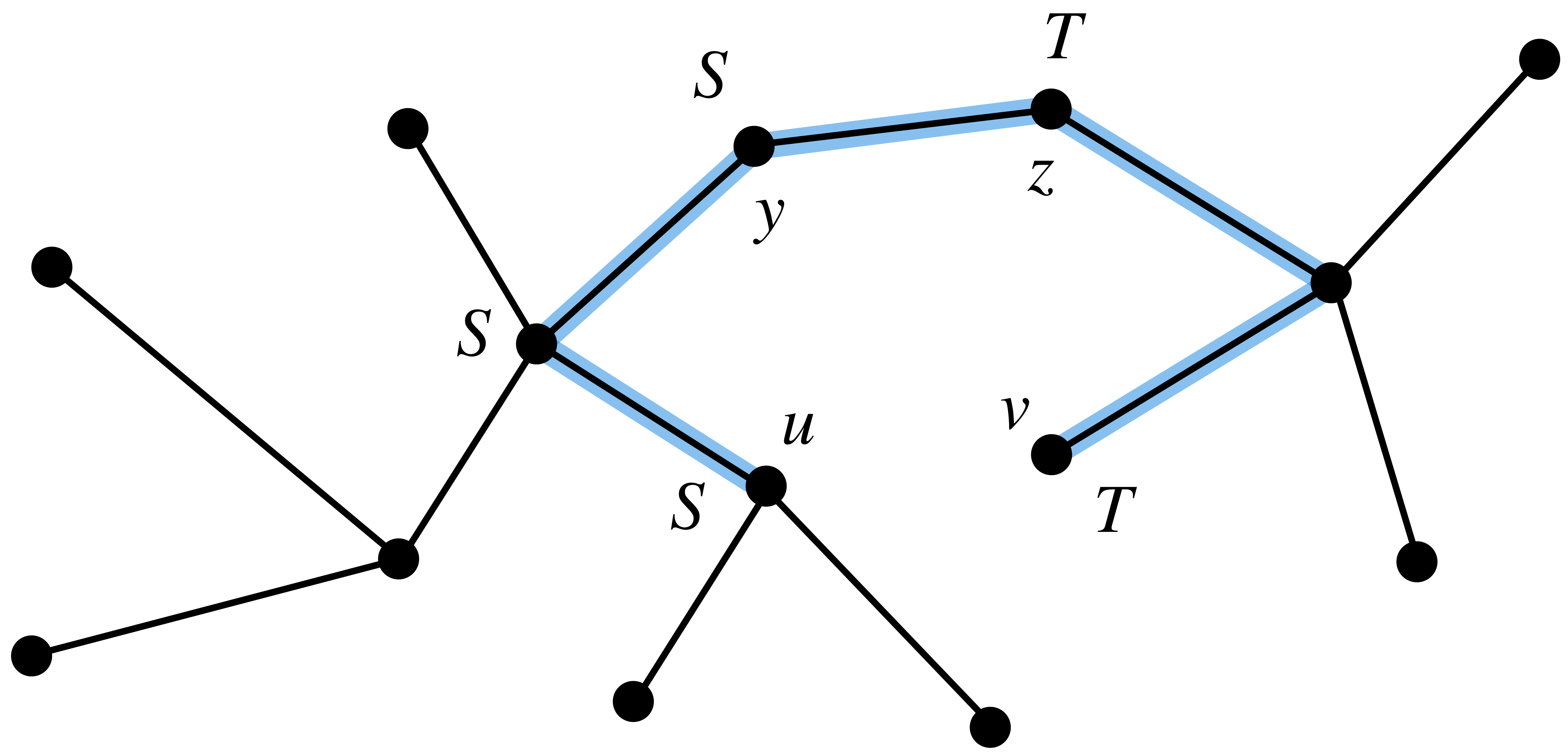

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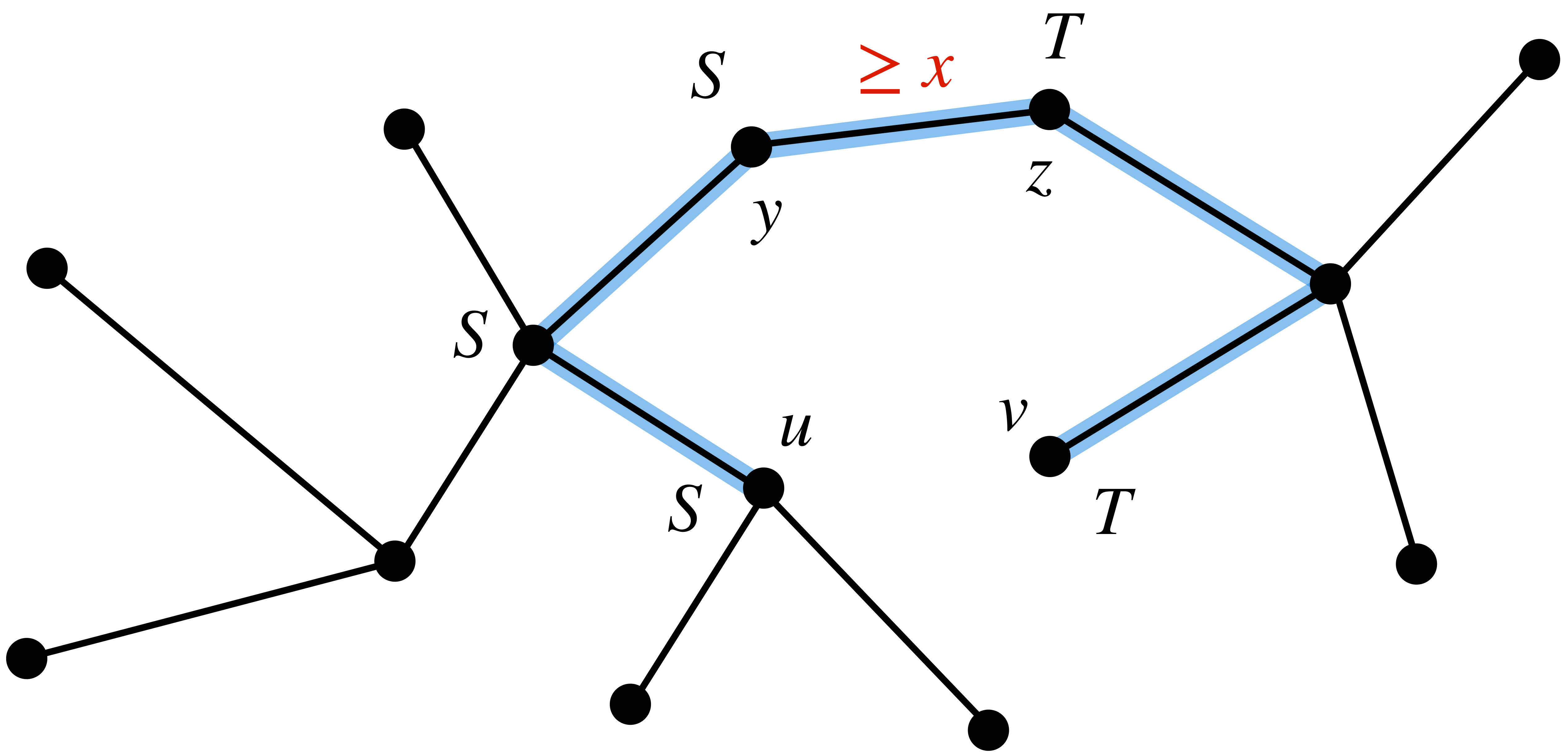

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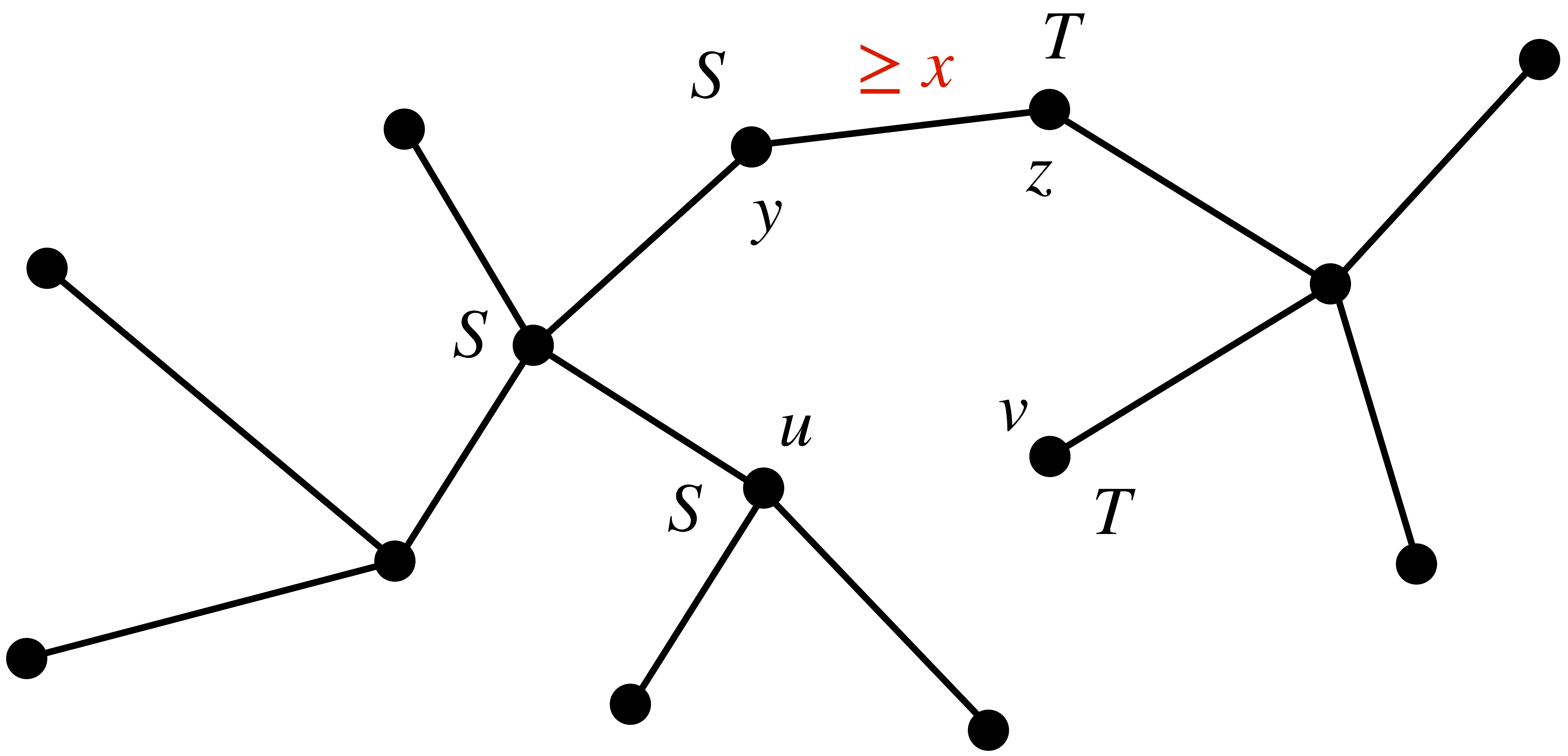

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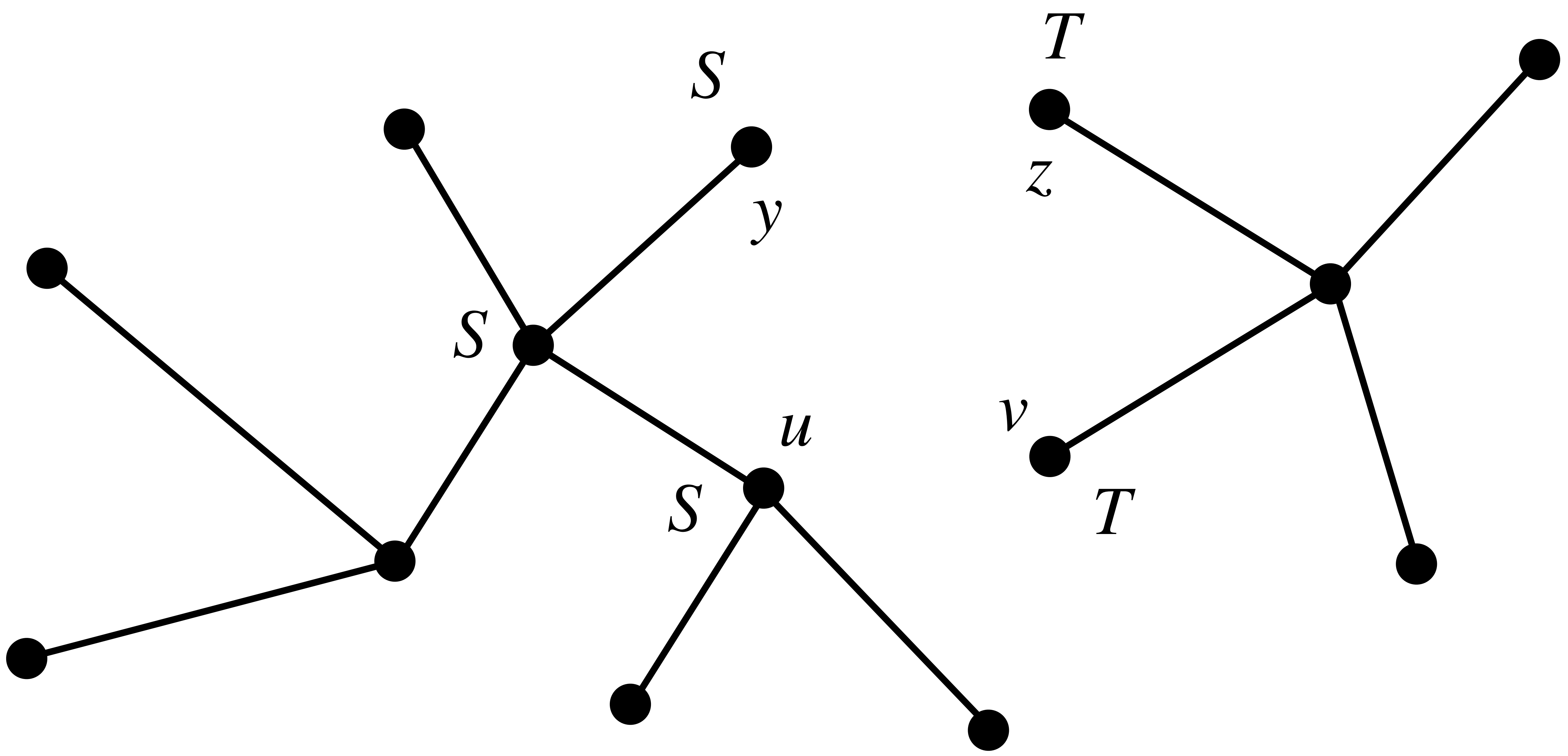

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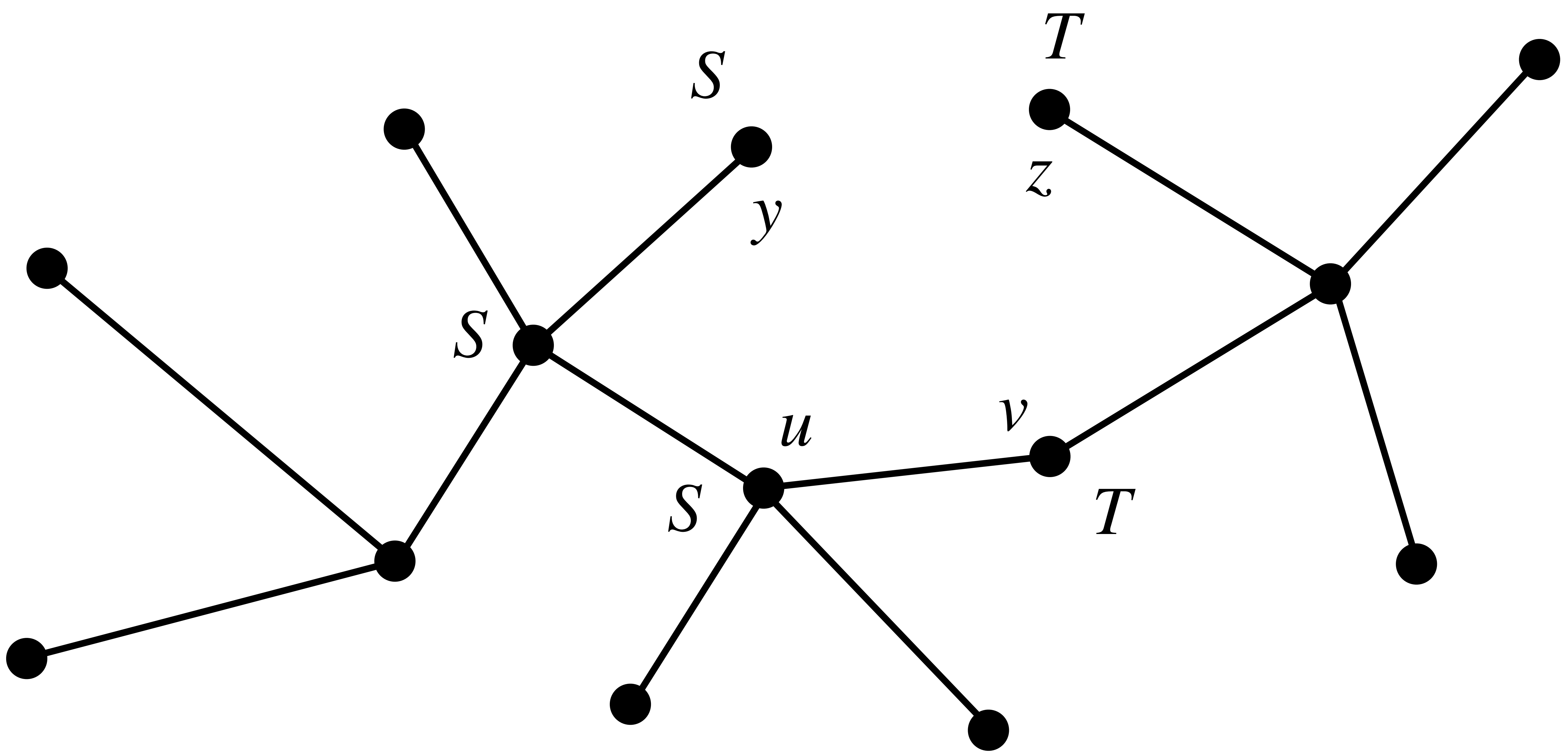

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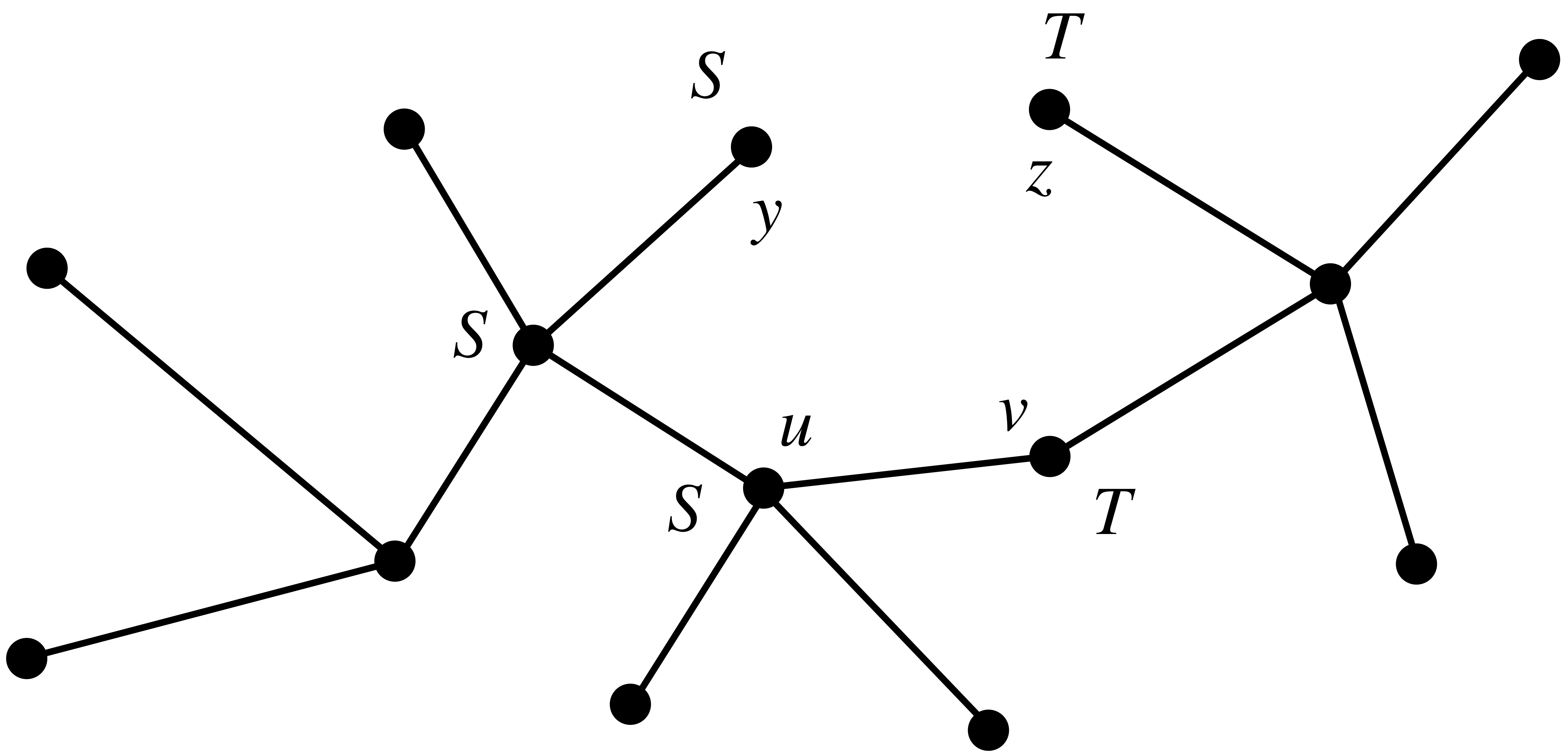

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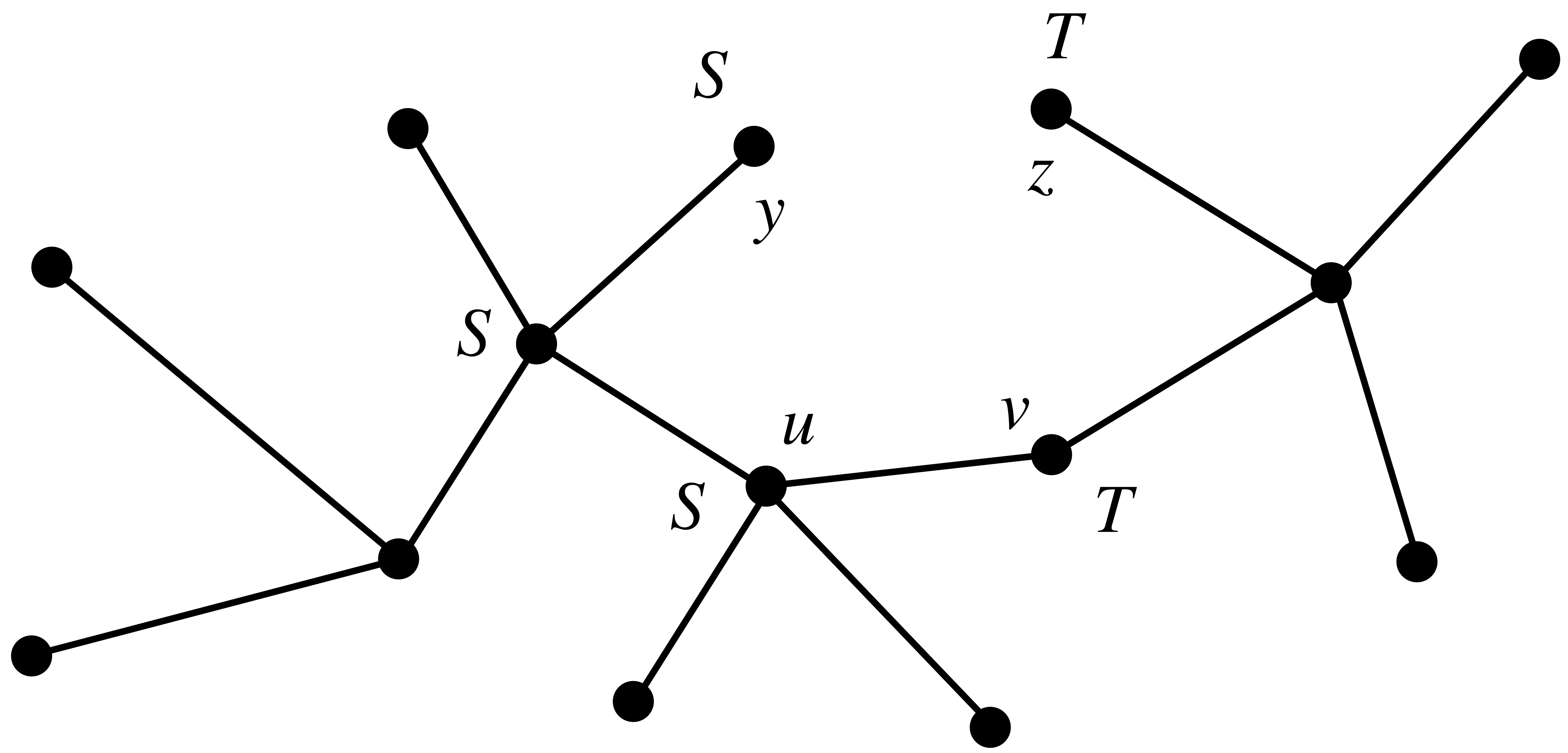

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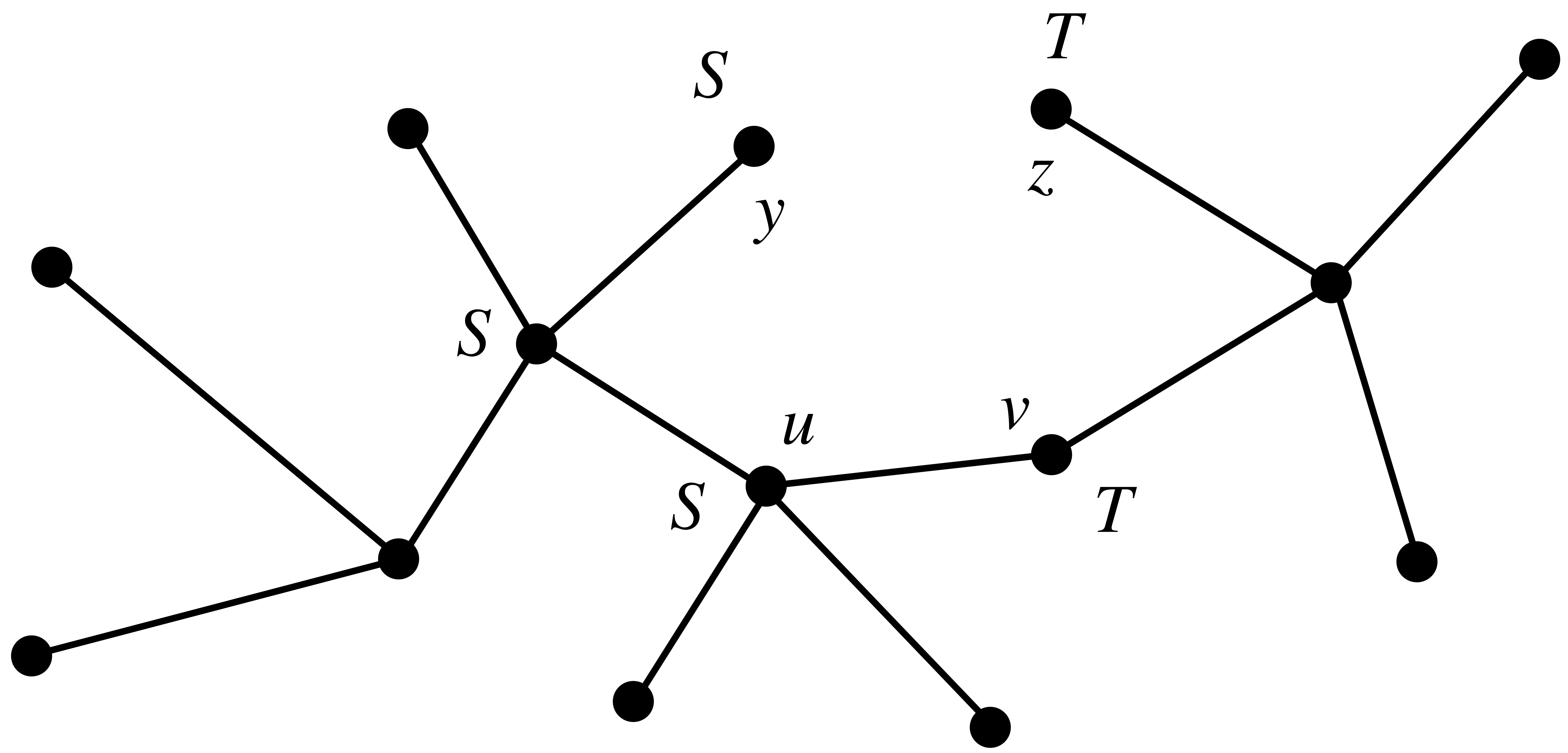

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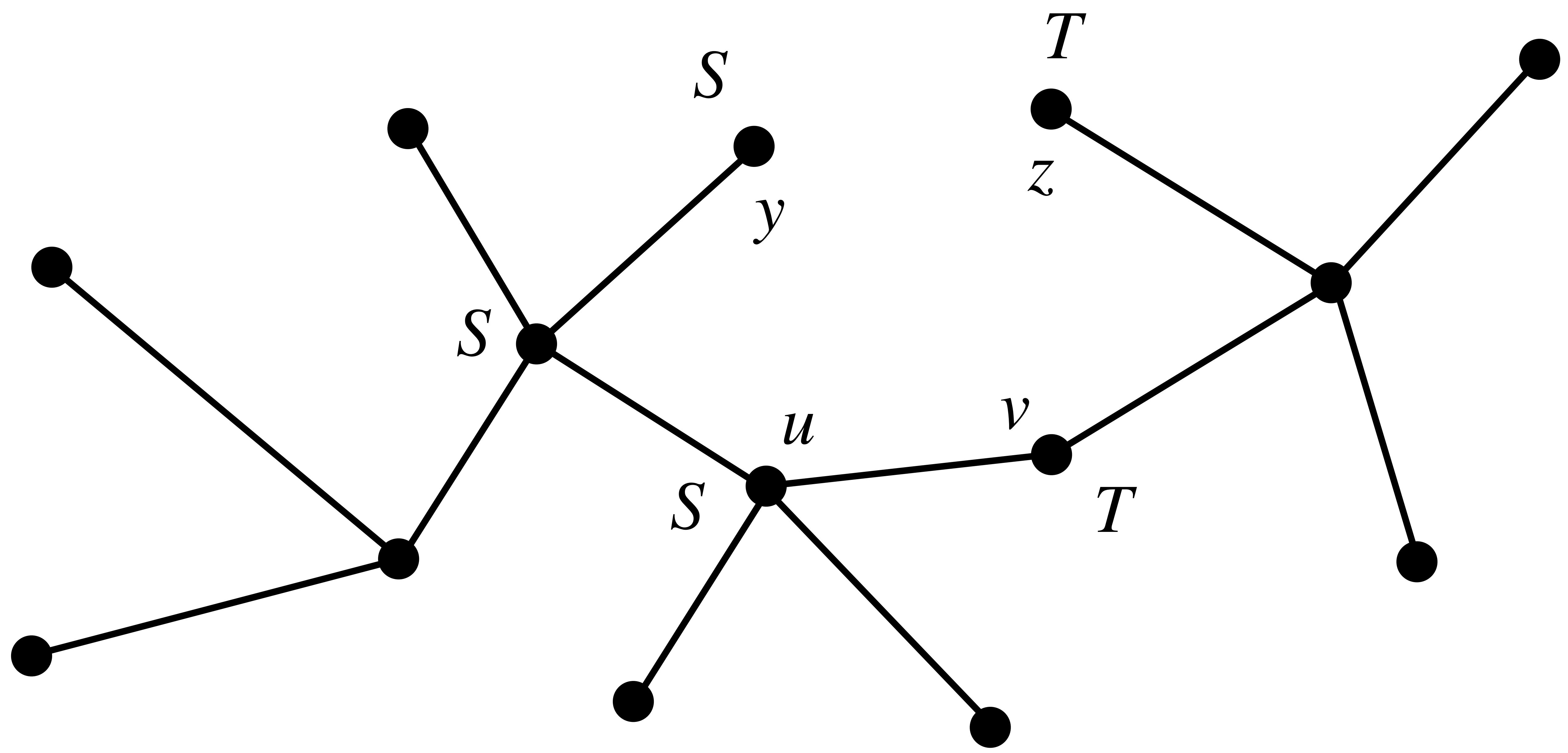

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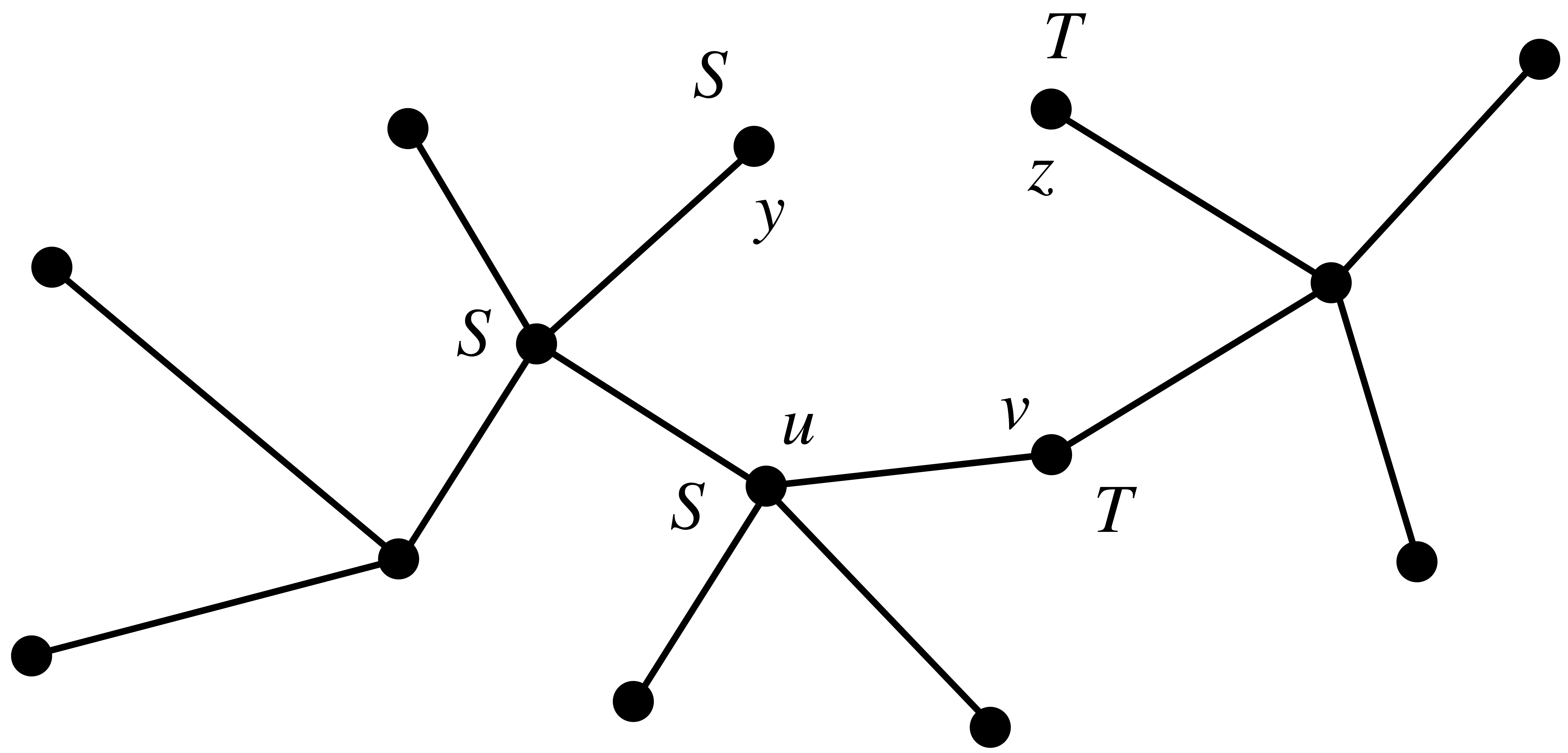

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